Levels of operator-valued R-transforms in free probability

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We consider the framework of the free probability theory of Voiculescu, where the algebras of (non-commutative) random variables are taken over a fixed algebra B, rather than over the field of complex numbers. As observed from the very beginning by Voiculescu, there are certain concepts from usual free probability which can be extended to the B-valued framework, and thus have a larger domain of applicability. Notably amomg them is the concept of R-transform (which plays the role of Fourier transform in free probability).

An exciting possibility offered by operator-valued free probability is the one of studying the connections between two levels of amalgamation, B and D, where D is a subalgebra of B. The talk will present a recent joint work of the speaker, D. Shlyakhtenko and R. Speicher, which studies this topic. Given a non-commutative random variable X, we will present a factorization condition which links the B-valued and the D-valued R-transforms of X, and which is equivalent to the fact that X is free from B with amalgamation over D. We also show how this factorization condition can be used when one studies in parallel the B-valued vs. the D-valued free Fisher information for X.