

Spectral Measures of the Almost Mathieu Operator

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On $\ell^2(\mathbf{Z})$ let $H_{\theta,\psi}$ be the operator

$$H_{\theta,\psi}(\xi)(n) = \xi(n-1) + \xi(n+1) + 2\cos(2\pi n\theta + \psi)$$

for $0 \leq \theta \leq 1$ and $0 \leq \psi \leq 2\pi$. In 1976 D. Hofstadter showed how to arrange the sets $\sigma_\theta = \cup_\psi \text{Sp}(H_{\theta,\psi})$ to form what is now called Hofstadter's butterfly. The main point of Hofstadter's paper was to describe the recursive structure of the picture, however he was unable to give a precise formulation.

We shall show how to find the spectral measures of the σ_θ 's relative to an integrated density of states and use this to make precise part of Hofstadter's programme. The main technical tool will be the realization of the rotation C*-algebras as a continuous field of C*-algebras.