U_k-admitting dcpos and the largest tcc subcategories of domains: two topological problems in Domain theory

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Domain theory is an area of the theoretical computer science, which uses domains, i.e., certain directed complete partial ordered sets (dcpos in short) as mathematical models to assign meanings to programs written in high-level programming languages. In this paper, we investigate two topological problems arising from Domain theory. We provide an example to show that U_k-admitting dcpos need not be sober via the Scott topology and prove that for a locally compact dcpo, U_k-admitting is equal to sober, which answers an open question posed by Heckmann in 1990. Continuous dcpos as the most important class of domains can also be viewed as topological spaces via the Scott topology, i.e., every continuous dcpo with the Scott topology is just a sober space such that the lattice of Scott open sets is a completely disributive lattice. It turns out that the category of FS-domains (uniformly approximated spaces called by Jung) is the largest topological cartesian closed (tcc in short) subcategory of the one of continuous dcpos, in sense of which is closed under products and fuction spaces with respect to the Isbell topology. This gives a partial answer of an open problem posed by Lawson and Mislove in 1990.