Self-similar flows of immiscible fluids

Brian Seymour
seymour@math.ubc.ca
UBC

Exact analytical representations are obtained describing self-similar unsteady flows of multi-phase immiscible fluids in the vicinity of non-circular, but constant strength, fronts. They describes flows in a porous medium that are produced when a stationary fluid is displaced by injecting another immiscible fluid into a region that is small compared with that occupied by the resident fluid. The analysis is based on a model of the "secondary recovery" process that is used in the petroleum industry to drive oil towards a recovery well by injecting water. The governing equations form a system of coupled first order partial differential equations in space and time of convective-diffusive type for the variations in the saturations and partial pressures. When the flows are unsteady and not uni-directional, only in the limiting case when the total mobility is constant and capillary effects can be neglected are there any known exact analytical solutions to the governing equations. Usually, solutions are sought by using numerical procedures. However, because of the nature of the governing equations and the existence of fronts whose locations must be determined as part of the solution, these procedures remain unreliable: changing the grid orientation can produce completely different numerical outputs. However, even though the flows are unsteady and two dimensional, the representations are obtained by using hodograph techniques.