Weak interaction limits for one-dimensional polymers

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Weakly self-avoiding walk is obtained by giving a penalty for every self-intersection to simple random walk path. The Edwards model is obtained by giving a penalty proportional to the square integral of the local times to the Brownian motion path, thereby also reducing the amount of time Brownian motion spends in self-intersections. We study these models in dimension one.

We prove that as the self-repellence penalty tends to zero, the large deviation rate function of the weakly self-avoiding walk converges to the rate function of the Edwards model. This shows that the speeds of one-dimensional weakly self-avoiding walk (if it exists) converges to the speed of the Edwards model. The results generalize results earlier proved only for nearest-neighbor simple random walks to general step-distributions for which the moment generating function exists. Moreover, the proof only uses weak convergence together with properties of the Edwards model, avoiding the rather heavy functional analysis that was used previously. Extending the previous proof seems hopeless.

The method of proof is quite flexible, and also applies to the strictly self-avoiding case where the variance diverges, under a certain technical condition on the Edwards model. This condition has not yet been proven, but we believe it to be true. This result proves an old conjecture by Aldous. Also, we can use it to prove that weakly self-avoiding walk with nearest neighbor attraction scales to the Edwards model for the appropriate choices of the parameters.

This is joint work with Frank den Hollander and Wolfgang Koenig.