Global existence and blow-up for reaction-diffusion equations

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In this paper, we consider a nonlinear reaction-diffusion equations as follows

$$u_t - \Delta u = u^{m_1} v^{n_1} w^{l_1}, (x,t) \in \Omega \times (0,\infty) \quad (1)$$
$$v_t - \Delta v = u^{m_2} v^{n_2} w^{l_2}, (x,t) \in \Omega \times (0,\infty) \quad (2)$$
$$w_t - \Delta w = u^{m_3} v^{n_3} w^{l_3}, (x,t) \in \Omega \times (0,\infty) \quad (3)$$
$$u(x,t) = v(x,t) = w(x,t) = 0, (x,t) \in \partial\Omega \times (0,\infty) \quad (4)$$
$$u(x,0) = u_0(x), v(x,0) = v_0(x),\ w(x,0) = w_0(x),\ x \in \Omega \quad (5)$$

where Ω is a bounded domain in R^n with smooth boundary $\partial\Omega, m_1, n_2, l_3 \geq 0, n_1 + l_1, m_2 + l_2, m_3 + n_3 > 0$ which ensure the system is completely coupled, and $u_0(x), v_0(x), w_0(x)$ are nonnegative continuous and bounded functions. This system arises in modelling heat propagation in a three-component combustible mixture. In this case u, v and w represent the tempretures of the interacting components, thermal conductivity is supposed constant equal for the three substances, and a volume energy release given by some powers of u, v and w is assumed.

Denote

$$D = (a_{ij})_{3\times 3} = \begin{pmatrix} (m_1 - 1) & n_1 & l_1 \\ m_2 & (n_2 - 1) & l_2 \\ m_3 & n_3 & (l_3 - 1) \end{pmatrix}$$

and

a_{ij}^* is the algebraic component of a_{ij}.

In this paper we will prove that

(1) If $m_1 < 1$, $(l_3 - 1)^* > 0$ and $n_3^* l_2^* -_2 -1)^*(l_3 - 1)^* \leq 0$, then all the nonnegative solutions of (1.1)-(1.5) are global.

(2) Under either of the following conditions, there are both global solutions and solutions which blow up in a finite time depending on the magnitude of the initial values.

(a) $m_1 < 1$, $(l_3 - 1)^* > 0$, and also $n_3^* l_2^* - (n_2 - 1)^*(l_3 - 1)^* \geq 0$.

(b) $m_1 \leq 1$, $(l_3 - 1)^* \leq 0$,

(c) $m_1 \geq 1$, $n_2 \geq 1$, $l_3 \geq 1$.

Here we give a complete picture of (1.1)-(1.5) for global existence and blow up in finite time.