Duality in Random Matrices and Biorthogonal Polynomials

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Correlation functions and spacing distributions in matrix models may be computed as determinants involving "integrable" Fredholm kernels. In the case of 2-matrix models with probability measures of the form of exponentials of traces of polynomials in the matrices, these kernels may be expressed by a generalized Christoffel-Darboux formula consisting of finite sums over sequences of biorthogonal quasi-polynomials and their Fourier-Laplace transforms. These give rise to representations of the Heisenberg commutation relations for the shift operators as finite band semi-infinite matrices having band sizes equal to the degrees of the polynomials defining the measure. These representations may be expressed, using the recursion relations, in terms of the biorthogonal quasi-polynomials within a finite sequence corresponding to the band size. This determines "dual pairs" of covariant derivative operators involving matrices having the size of the band in one of the shift operators, with entries that are polynomials of degree equal to the size of the band of the dual operator. Interchanging the two, it is shown that the resulting characteristic polynomials are identical. Deforming the measure within this class gives rise to commuting flows that preserve the (generalized) monodromy data of both the operators. The isomonodromic tau function associated to this class of operators is related to the partition function for the matrix ensemble.

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