

Hall algebras and quantum groups

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By a result of Kac (after Gabriel), there exists a nice correspondence between the indecomposable representations of a quiver and the positive roots of the Kac-Moody Lie algebra. In 1990, C.M.Ringel constructed an associative algebra, called Hall algebra, from the representation category of the quiver, to which an explicit Hopf algebra structure is endowed and its composition subalgebra is showed to be a realization of the quantum group in the sense of Drinfeld-Jimbo. Recently, by Sevenhant-Van den bergh, it is known that the whole Ringel's Hall algebra is in fact isomorphic to the quantized enveloping algebra of a generalized Kac-Moody algebra in the sense of Borchers. A complete proof of this result can be found in my joint work with B.Deng (to appear in J.Algebra). In my talk, I will tell this story briefly and point out some interesting consequences. In particular, a new proof of a weak form of the Kac theorem and a Hall algebra interpretation to a conjecture of Kac will be presented.