The Nonlinear Schrödinger equation: Self-focusing and Wave Collapse

Catherine Sulem
sulem@math.toronto.edu
University of Toronto

The nonlinear Schrödinger (NLS) equation provides a canonical description for the envelope dynamics of a quasi-monochromatic plane wave propagating in a weakly-nonlinear dispersive medium. I shall concentrate in this talk on the phenomenon of *wave collapse* which refers to solutions of the NLS equation (or of its generalizations involving couplings to other fields) whose amplitude blows up in a finite time. The features of the singularity are important because they reflect the onset of violent phenomena arising on the primitive equations even if in this context, the collapse is eventually arrested by additional effects that become relevant after sufficiently small scales have been formed.

Several approaches will be reviewed, ranging from rigorous mathematical analysis to formal asymptotic expansions and numerical simulations in an attempt to put in perspective the rigorous theory of the NLS equation and the physical understanding of the wave-collapse phenomenon that plays an important role in plasma physics and nonlinear optics.