Periods of modular forms and rational points on elliptic curves

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We will discuss some questions arising from the study of two related questions: (1) the problem of finding explicit analytic constructions of class fields of number fields (known as Hilbert's 12th problem, or Kroneckers Jugendtraum) and (2) the problem of constructing rational points on (modular) elliptic curves. The main (and, for now, essentially the only) tools for tackling both problems are supplied by the theory of Complex Multiplication developped by Kronecker, Weber, Hasse, Deuring, and (more recently) Shimura and Taniyama. Unfortunately, this theory applies only to (certain) abelian extnesions of CM-fields. I will describe a conjectural generalisation of the theory of complex multiplication which applies to a larger class of fields and would allow, in particular, the construction of all class fields of real quadratic fields by analytic means.