

On the structure of ideals, K-theory of Roe algebras

Xiaoman Chen

xchan@fudan.edu.cn

Dept. Math., Fudan University

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As we all know, Baum-Connes and Coarse Baum-Connes Conjectures imply many geometric and topological applications. Isomorphism between K-homology of space and K-Group of C^* -algebra provides the way from analysis to topology. The key point is the calculation of K-group of C^* -algebra. Roe algebra (some kind of C^* -algebra) as non-commutative object plays a very important role. Recently using expander graph Higson and Guoliang Yu gave a counterexample of Coarse Baum-Connes which shows that the ideal generated by ghost elements is an obstacle. From the algebraic view we study the structure of ideal of Roe algebra. In particular bad behavior of ghost ideal will be shown. Cyclic cohomology of Smooth algebra of C^* -algebra can detect information of K-group of corresponding C^* -algebra. In case of polynomial growth one can build up smooth algebra of Roe algebra. The rapidly decay property of space guarantees the Schwartz space to be a smooth dense subalgebra. To be much more different with group C^* -algebra, we give necessary and sufficient conditions for the Schwartz space to be smooth dense subalgebra of Roe algebra. In order to attack the Coarse Baum-Connes conjecture, Guoliang Yu introduced in a localization Roe algebra. The local index map is shown to be an isomorphism for finite dimensional simplicial complex with the spherical metric. Motivated by Guoliang yus work and J. Roe and Higsons work of Paschke duality, we investigate the relation of localization algebras and K-homology groups. Some kind of local K-homology is defined which is shown to be isomorphism to the K-homology in case of the Rips complex of a proper metric space with bounded geometry. Then we obtain another obstruction group to the Coarse Baum-Connes conjecture.