FINAL REPORT ON THE CRG ON NUMBER THEORY: 2003–2005

Introduction.
Number theory is one of the oldest and deepest branches of modern mathematics. Its development has led to some of the most sophisticated and profound of mathematical ideas (e.g Wiles proof of Fermat's Last Theorem). And yet it remains broadly useful in many areas of pure and applied mathematics. Indeed, it is remarkable how often number theory comes to bear both in other areas of mathematics and in applications. Notable recent examples are the fields of cryptography and internet security whose protocols are based on computationally intractible number theoretic problems. Number theory is particularly strong in Canada with the PIMS Number Theory Group featuring prominently. This group is large and well distributed across the PIMS Universities. It has a number of prominent senior world-class researchers leading a group of richly talented young mathematicians.

PIMS support of the Number Theory group, especially the core group in Vancouver, has been a major factor in the expansion and strengthening of that group. The core group in Vancouver consists of four number theorists at SFU (P. Borwein, I. Chen, S. Choi and N. Bruin) and four number theorists at UBC (D. Boyd, M. Bennett, V. Vatsal and G. Martin). Prior to the formation of the CRG, S. Choi and N. Bruin had been joint SFU/UBC PIMS postdoctoral fellows and were subsequently hired into tenure-track positions at SFU.

The guaranteed funding of two PIMS postdoctoral positions during the CRG period allowed the group to attract Ben Green to Vancouver for a year of postdoctoral work as a PIMS postdoctoral fellow. His success during that year and his subsequent appointments as chair at Bristol and now as Hershel Smith Professor at Cambridge indicate the wisdom of that decision. One of our other PIMS postodoctoral fellows during that period, Friederich Littmann, found an academic position while another, Christopher Rowe, decided to change direction into financial mathematics. Even after the guaranteed funding for two fellows ended in April 2005, the critical mass of the group built up during the two years of extra funding was certainly a significant factor in being able to attract our current very strong group of PIMS PDFs,

Chris Sinclair, Matilde Lalín and Ronald van Luijk, who were awarded PIMS PDFs in competition with a very strong field of applicants.

Regular activities of our group that were in existence prior to the CRG and still continue are the joint PIMS sponsored SFU/UBC number theory seminar (held biweekly alternating between SFU and UBC) and the PIMS sponsored Pacific Northwest Number Theory seminar, which meets once each year at various locations in BC, Washington and Oregon. During the CRG, we were able to plan more ambitious events including a summer period of concentration on Mahler's measure held at SFU in the summer of 2003. One very successful event which certainly would not have taken place without the stimulus of the CRG funding was the one day conference for postdoctoral fellows in Number Theory held in Vancouver prior to the winter CMS meeting in 2003. This broadened the network of many young number theorists and has been repeatedly cited by many of them as a significant event in their career.

The SFU/UBC Number Theory group organized the PIMS sponsored international conference of the Canadian Number Theory Association in 2006. Again, this event did not occur during the CRG period but the network established during the period of the CRG was certainly a significant factor in its success. Other indications of the continuing influence of the two years of CRG funding are the number of successful BIRS proposals put forward by subsets of the group each year, for example the BIRS workshop *Number Theory Inspired by Cryptography*, Nov 5-10, 2005, which was conceived during the period of the CRG funding and the BIRS workshop *Analytic Methods for Diophantine Equations*, May 13–18, 2006, which built on the success of the two BIRS workshops in November, 2004 organized by the CRG during its period of funding.

No doubt the strength and cohesiveness of the core group of the CRG was a factor in its success in obtaining an NSERC Leadership grant of $40,000 per year for 2003–2007 which was used entirely for the support graduate students at SFU and UBC. This led to a considerable strengthening of the graduate program in Number Theory at SFU and UBC.

Faculty (Apr 1, 2003 – Mar 30, 2005).

- CRG Leaders: Peter Borwein (SFU), David Boyd (UBC)
- UBC: Michael Bennett, David Boyd, Bill Casselman, Rajiv Gupta, Izabella Laba, Greg Martin, Nike Vatsal
- SFU: Peter Borwein, Imin Chen, Stephen Choi, Petr Lisonek

- U. Alberta: James Lewis
- U. Calgary: Richard Guy, James Jones, Renate Scheidler, Hugh Williams
- U. Washington: Ralph Greenberg, Adrivan Iovita, Neal Koblitz, Boris Solomyak
- Other Insitutions: Amir Akbary (U. Lethbridge), Edward Dobrowolski (College of New Caledonia), Matt Klassen (DigiPen Inst. of Tech.), Kristin Lauter (Microsoft)

PIMS Postdoctoral Fellows (Apr 1, 2003 – Mar 30, 2005).

- Ben Green, Sep 2003 – Aug 2004, now Professor, Trinity College, Cambridge
- Friederich Littmann, Sep 2003 – Aug 2005, now Assistant Professor, U. North Dakota
- Christopher Rowe, Sept 2003 – Aug 2005, now in financial mathematics, Boulder, Colorado

PIMS sponsored events of CRG during the funding period.

- PIMS PNWNT conference, Seattle, Apr 5-6, 2003
- PIMS sponsored BIRS workshop, *The Many Aspects of Mahler's Measure*, Apr 26 – May 1 , 2003.
- PIMS Number Theory Group Thematic Summer Program, *Mahler Measure*, Jun 6-29, 2003.
- PIMS Distinguished Visitor, Jeff Vaaler, Jun 6-29, 2003.
- PIMS sponsored BIRS workshop, *Current trends in arithmetic geometry and number theory*, Aug 23–28, 2003.
- PIMS CRG in Number Theory, postdoctoral conference, Vancouver, Dec 5, 2003.
- PIMS PNWNT conference in Corvallis, Oregon – Apr 17, 2004
- PIMS Distinguished Visitor, Sergei Konyagin, Mar 1 – May 1, 2004.
- PIMS sponsored BIRS workshop, *Explicit Methods in Number Theory*, Nov 13–18, 2004.
- PIMS sponsored BIRS workshop, *Number Theorists weekend*, Nov 18–20, 2004.
- PIMS sponsored BIRS workshop, *Diophantine approximation and analytic number theory*, Nov 20–25, 2004.

Some Significant Achievements of the PIMS CRG in Number Theory during 2003-2005 and subsequently.

- The PIMS PDF Ben Green (2003-04) was a spectacular sucess. During his time in Vancouver he and Terry Tao resoved an important old problem that shows that the sequence of primes contains arbitrarily long arithmetic progressions. This work won him a Clay Research Prize for in 2004, led to his appointment to a chair at Bristol University and then his appointment as Hershel Smith Professor of Pure Mathematics and Fellow of Trinity College, Cambridge. This work also led to his invitation to speak at the International Congress of Mathematicians in Madrid in August 2006. (This work was also cited in the Fields Medal awarded to Terry Tao at that same congress).

- CRG member Nike Vatsal was awarded the André Aisenstadt prize of the CRM in 2004, the Ribenboim prize of the CNTA in 2006 and was given an invitation to speak at the ICM in 2006 as the result of his groundbreaking work on the distribution of Heegner points.

- The BIRS workshop in Mahler measure and the subsequent summer program at SFU in 2003 were influential in attracting two of our current postdoctoral fellows, Chris Sinclair and Matilde Lalín. European participants in these events, Vincent Maillot and Marie-José Bertin were inspired to organize a workshop at CIRM in June 2005 on Mahler measure and then Dan Silver and Susan Williams organized a conference on Mahler measure and Knot Theory at the University of South Alabama in January 2006. A number of successful collaborations were begun during the events of 2003, e.g the work of P. Borwein, E. Dobrowolski and M. Mossinghoff on Lehmer's problem for polynomials with odd coefficients.

- The BIRS workshop on Explicit Methods in Number Theory in November 2004 was the scene of the solution of a long-standing problem in the theory of diophantine equations, namely the complete solution of the equation $x^2 + y^3 = z^7$ in relatively prime integers, due to B. Poonen, M. Stoll and E. Schaefer. Some of the organizers of that event have been successful in another bid for a BIRS workshop on *Explicit methods for rational points on curves*, Feb 4-9, 2007, aimed at a deeper understanding of Kim's new *non-abelian Chabauty* method. The workshop and the subsequent activity in the field also played a role in

attracting one of our current PIMS PDFs in Number Theory, Ronald van Luijk, who will join us in January 2007.