Report on Period of Concentration 2003-2005 of the Collaborative Research Group in Dynamics

The principal events during the period of concentration were:

- Distinguished Chair, Klaus Schmidt (University of Vienna), November 2002. Klaus Schmidt is one of the world leaders in ergodic theory and especially algebraic dynamics, a subject in which he literally wrote the book, an award winning research monograph laying the foundations of the subject. Dr. Schmidt gave series of lectures on the basics and most recent developments in both Victoria and Seattle, as well as a colloquium talk at U.B.C.

- Banff International Reseach Station 5-day Workshop on Joint Dynamics, June 28-July 3, 2003. In classical dynamical systems, the evolution of a physical system is one dimensional, modelled on temporal evolution. The relatively recent development of dynamics in higher dimensions has revealed a number of striking and unexpected results. The dynamics group within PIMS has been one of the world centres for this development. The workshop brought together most of the leaders of this field.

- Banff International Reseach Station 5-day Workshop on The Many Aspects of Mahler's Measure, April 26-30, 2003. One area where higher dimensional dynamics has been particularly effective and important has been in algebraic dynamics. This field has seen a beautiful development which includes interactions between dynamics, algebra and number theory. The University of Washington has been a world leader is this program. This workshop concentrated on aspects in this area, especially those related to number theory. It involved a unique mix of people from these different areas.

- Banff International Reseach Station Research in Teams on Topological Orbit Equivalence, April 26- May 10, 2003. The participants, Giordano

(Ottawa), Putnam (Victoria) and Skau (Trondheim) have developed a program extending the notion of orbit equivalence to the topological setting. This is an appealing mix of ideas from dynamics and operator algebras. This particular project involved extending dynamical ideas from one dimension to higher ones. The result gave conclusive results for certain special cases and has lead to a recent solution to the problem in dimension two.

- Banff International Reseach Station 5-day Workshop on Aperiodic Order: Dynamical Systems, Combinatorics and Operators, May 29- June 3, 2004. Aperiodic order is the phenomenon of physical and mathematical geometric structures which display a high degree of regularity but are not periodic. It has been an extremely lively area since the discovery of such mathematical objects in the 1960's and quasicrystals (analogous physical materials) in the 1980's. It is also notable for the diversity of mathematics that it draws upon to quantify and explain the nature of the order. It may seen viewed as dynamics as a 'spatial' evolution. The Universities of Washington, Alberta and Victoria are all well-represented in various aspects of this field making PIMS a unique world centre on the area. This workshop was another in a series usually held at research institutes in Europe bringing together experts on a wide variety of areas.

- Distinguished Chair, Dan Rudolph (University of Maryland and Colorado State University), October 2004. Dan Rudolph is another world leader in ergodic theory, known especially for his work on restricted orbit equivalence. His lectures in Victoria on the subject were an excellent introduction to the subject, leading to the most significant results. The visit was especially fruitful since Dr. Rudolph is beginning to extend these techniques to the topological setting where Putnam and his collaborators have already obtained interesting results. This stimulated some very interesting interactions which will likely be pursued over the next few years between Colorado State and Victoria. The visit was extremely useful for graduate students because of Dr. Rudolph's stimulating style of presentations, the interest in the research program and a special informal session in which he participated on 'how to be a graduate student'.

- Banff International Reseach Station Focused Research Group on Topo-

logical Aspects of Aperiodic Order, July 16-30, 2005. As noted above, the area of Aperiodic Order has drawn on a wide variety of mathematics. Two of these are the areas of topology and operator algebras. Here, classical invariants from topology and others from non-commutative geometry are used to describe and quantify aperiodic order. This particular activity had a unique mix of people with backgrounds in topology, operator algebras and physics. Several open problems were solved during the course of the two weeks. In addition, a volume, to be written by the participants and published by the American Mathematical Society, is planned as a kind of handbook for the subject.

- Summer School in Aperiodic Order, August 2005, University of Victoria. The summer school was a highly successful event, intended mainly for graduate students and post-doctoral fellows. There were four lecture series covering the very broad range of the basics of the subject: harmonic analysis for discrete point patterns, ergodic theory, physics and topological aspects. There was also the opportunity for short presentations by participants on their own work.

- Second Northwest Dynamical Systems Symposium, August 2005, University of Victoria. This was a follow-up to a similar meeting held five years earlier. Held the week after the Summer School, there was a large overlap among the participants. But this meeting also attracted ergodic theory experts from across North America and Europe. These meetings have helped establish Victoria as a centre for dynamics.

The PIMS faculty which were directly involved in the CRG:

- University of Victoria: Chris Bose, Anthony Quas, Ian Putnam.
- University of Washington: Doug Lind, Selim Tuncel, Boris Solomyak.
- University of British Columbia: Brian Marcus, Richard Kenyon.
- University of Alberta: Robert Moody (now at Victoria).

The University of Washington has long been a leading centre for ergodic theory and dynamical systems. Within the CRG, it assumed a natural leadership rôle. However, the growth of dynamics in the PIMS sites in Canada has been truly remarkable over the last five years. It includes the appointments

of Brian Marcus and Rick Kenyon (Canada Research Chair) at U.B.C. and Anthony Quas (Canada Research Chair) at U.Vic. Through these appointments and the Period of Concentration, the region now has a large presence on the dynamics scene. This has also seen a growth in the number of graduate students and post-doctoral fellows. One, Wael Bahsoun, who was a PIMS PDF through the CRG at Victoria, now has a position at the University of Manchester. Two other former Ph.D. students of Robert Moody at Alberta have held or now hold NSERC PDF's in the CRG. Jeongyup Lee spent one year at the University of Washington and the second at the University of Victoria. Nicolae Strungaru is now at Victoria. Another development is the move of Yuval Peres to Microsoft Research in Seattle. Already, he is involved in a collaboration with Doug Lind and Boris Solomyak at UW.

The CRG has been very effective in establishing links between the sites. There has been some sharing of visitors. In the area of aperiodic order, it has been critical in making connections between Moody's group at Alberta and those at Washington, most notably Boris Solomyak. Solomyak and Lee now have a strong collaboration which has made significant advances over the past two years. In the area of algebraic dynamics, one of the main outstanding problems is the so-called Furstenberg conjecture. Although it remains open, significant progress was made in collaborations between Manfred Einsiedler, Klaus Schmidt and Elon Lindenstrauss. These were largely made possible because of events in the Period of Concentration.