Banff International Research Station
2006 Proposal Review

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06w5030: Schrödinger Evolution Equations

Organizers:

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Objectives:

The recent work on variable coefficient dispersive estimates and almost conservation laws and their applications has been carried out by several teams working in isolation from one another. A main objective is to bring together, among other allied researchers, the quintet (Colliander, Keel, Staffilani, Takaoka, Tao) who have been focusing on well-posedness and scattering for NLS; a diverse group from Paris (Burq, Carles, Gerard, Tzvetkov, Planchon, Robbiano, Zuily) whose work has been on well-posedness for NLS and associated multilinear estimates via semiclassical methods, as well as Strichartz estimates for variable-coefficient problems; several Japanese researchers (Chihara, Doi, Nakamura, Yajima) who have concentrated on propagation of singularities, dispersive smoothing effects, and parametrices for the linear problem; a group who have been studying fully nonlinear problems of Schr\"odinger type (Kenig, Ponce, Vega).

These groups, while aware of each others' work, have had little chance to interact in an intensive, structured way. Further interaction between groups using microlocal and geometric techniques developed for linear PDE, those using harmonic analysis tools, and those using methods adapted for rough coefficients and highly nonlinear PDE, will allow the central problems in the field to be viewed more synergetically. The state of the art of research into NLS has become rather difficult for a single mathematician to grasp, as phenomena split into a wealth of special cases depending on choice of dimension, regularity, nonlinearity, and behaviour of coefficients. A workshop will offer participants a much-needed broad view of this growing field. A better, more systematic view of the fundamental well-posedness questions, as well as of more refined phenomenology such as scattering and structure of blowup solutions, is bound to emerge.

Subjects:

Subject 1: Partial differential equations
Subject 2:

Overview:

Significant research activity in recent years has been focused on Schr\"odinger evolution equations, which appear as canonical models of wave behavior in diverse physical settings. Dispersive estimates provide the size and smoothness control in spacetime on linear Schr\"odinger evolutions and are the basic tools for constructing local-in-time solutions of nonlinear problems using fixed point and other methods. Conservation laws give a priori control on solutions which may be used to investigate their maximal-in-time behavior. Dispersive estimates and conservation laws combine to give a rather complete theory of certain simpler Schr\"odinger evolution equations with smooth enough initial data. Important variable coefficient problems, emerging from geometry, soliton stability, inhomogeneous wave media,
and involving large and/or rough potentials and rough initial data, can not be treated directly and have inspired recent activity toward the development of more flexible techniques. Many of the recent developments provide new flexible ways of understanding DISPERSION and CONSERVATION properties of Hamiltonian Schr\"odinger evolutions.

Variable Coefficient Dispersion

In linear Schr\"odinger evolution, waves decrease in amplitude and spread out in space as time progresses; this phenomenon is called dispersion. In the setting of constant coefficient linear Schr\"odinger equations, there are formulae expressing the solution in terms of the initial data. Stationary phase, functional analysis and direct calculation techniques based on the explicit representations of the solution have been used to prove dispersive estimates, such as local smoothing and Strichartz estimates, which quantify the behavior of linear constant coefficient Schr\"odinger waves. There are no complete representations of solutions in terms of initial data for linear equations with variable coefficients. To obtain dispersive estimates for variable coefficient setting therefore requires that new proofs, not based upon solution formulae, be devised. New approaches to proving dispersive estimates based upon virial-type identitities, and microlocal/semiclassical analysis have been developed and, in some situations, have produced optimal results.

Almost Conservation

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In simplified situations, such as for certain equations with constant coefficents and a dilation invariance, almost conservation laws have been introduced to capture aspects of conservative behavior even when the solutions under consideration are too rough to define the associated conserved quantity. These ideas have led, in some cases, to an optimal treatment of initial value problems and to new detailed descriptions of the time asymptotic behavior of rough solutions. Integral conserved quantities can also be expressed differentially: using the equation, the time derivative of the conserved density is shown to be a divergence of an associated flux. Study of the flux term then provides the insights into how and why there is conservation which permits proving useful almost conservation properties. The multilinear estimates underpinning the proofs of almost conservation laws to date have relied upon explicit representations of linear dispersion. The implementation of almost conservation methods in nonlinear variable coefficient Schr\"odinger evolutions requires new ideas to exploit the recent microlocal/semiclassical analysis based proofs of the dispersive properties of these flows.

Participants:

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