Fresnel theory in radar wave propagation problem- A better alternative to rays

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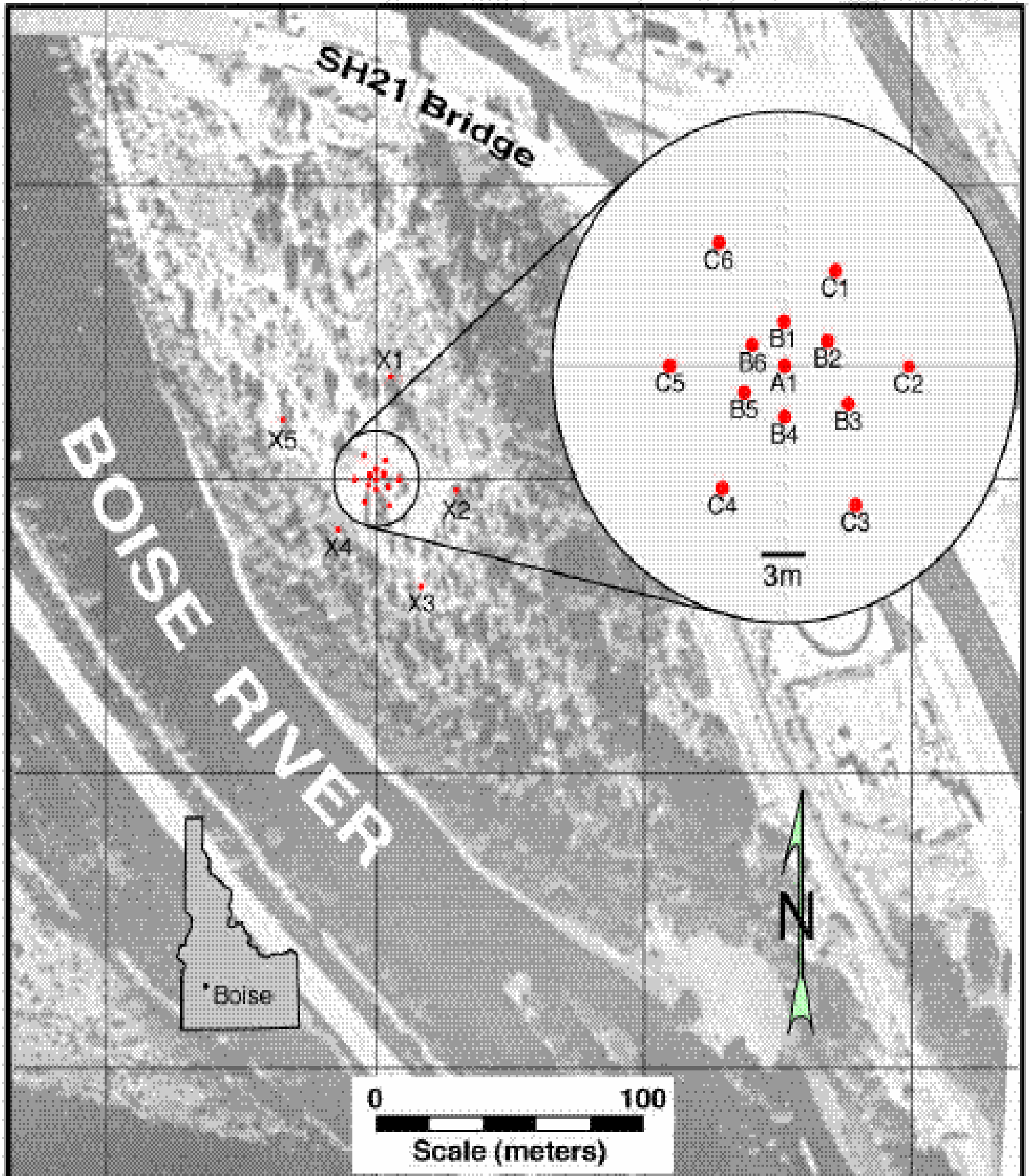
Motivation- Time Lapse Tracer Test

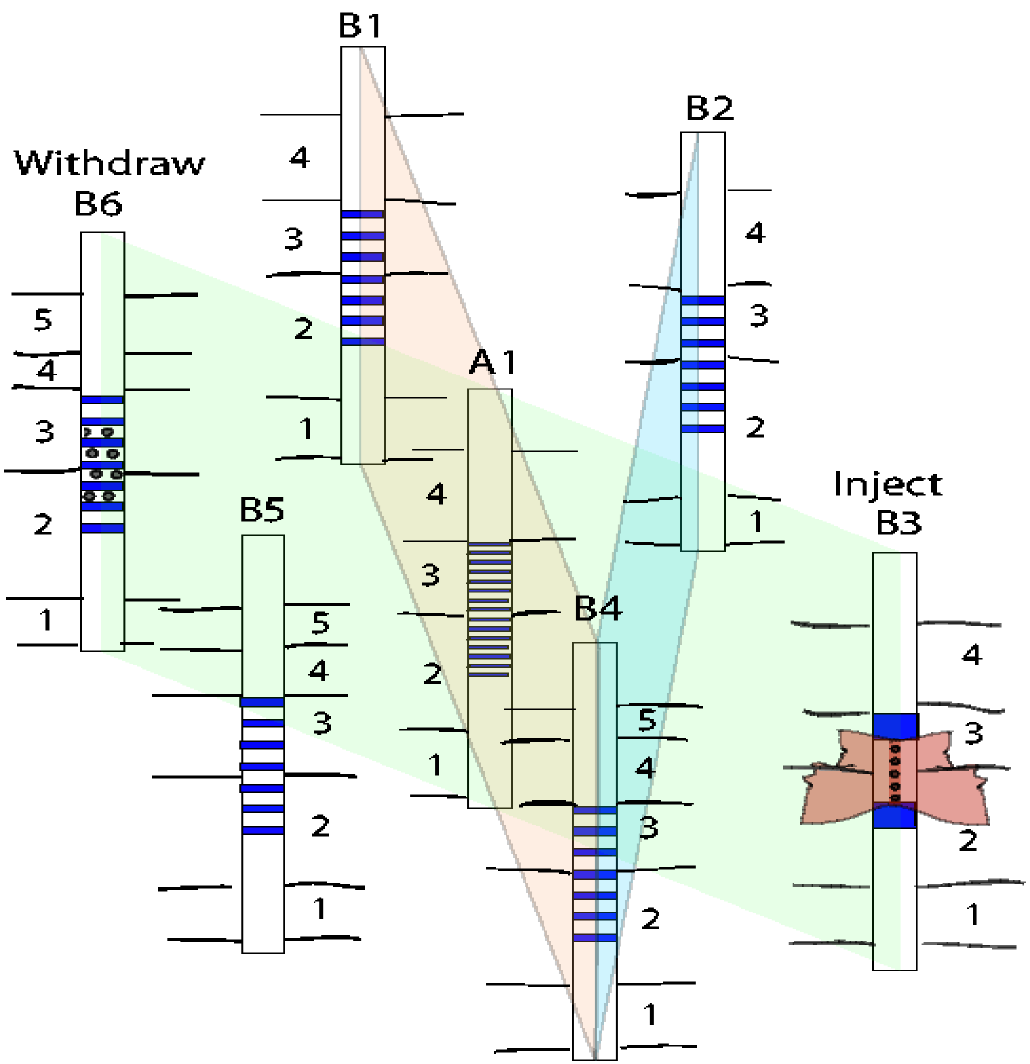


- Boise Hydrogeophysical Research Site (BHRS)
- Well Field Test Site in a gravel bar close to Boise river
- 18 fully penetrating wells in fluvial unconfined aquifer
- Aquifer is ~20m deep

Outcrop Heterogeneity

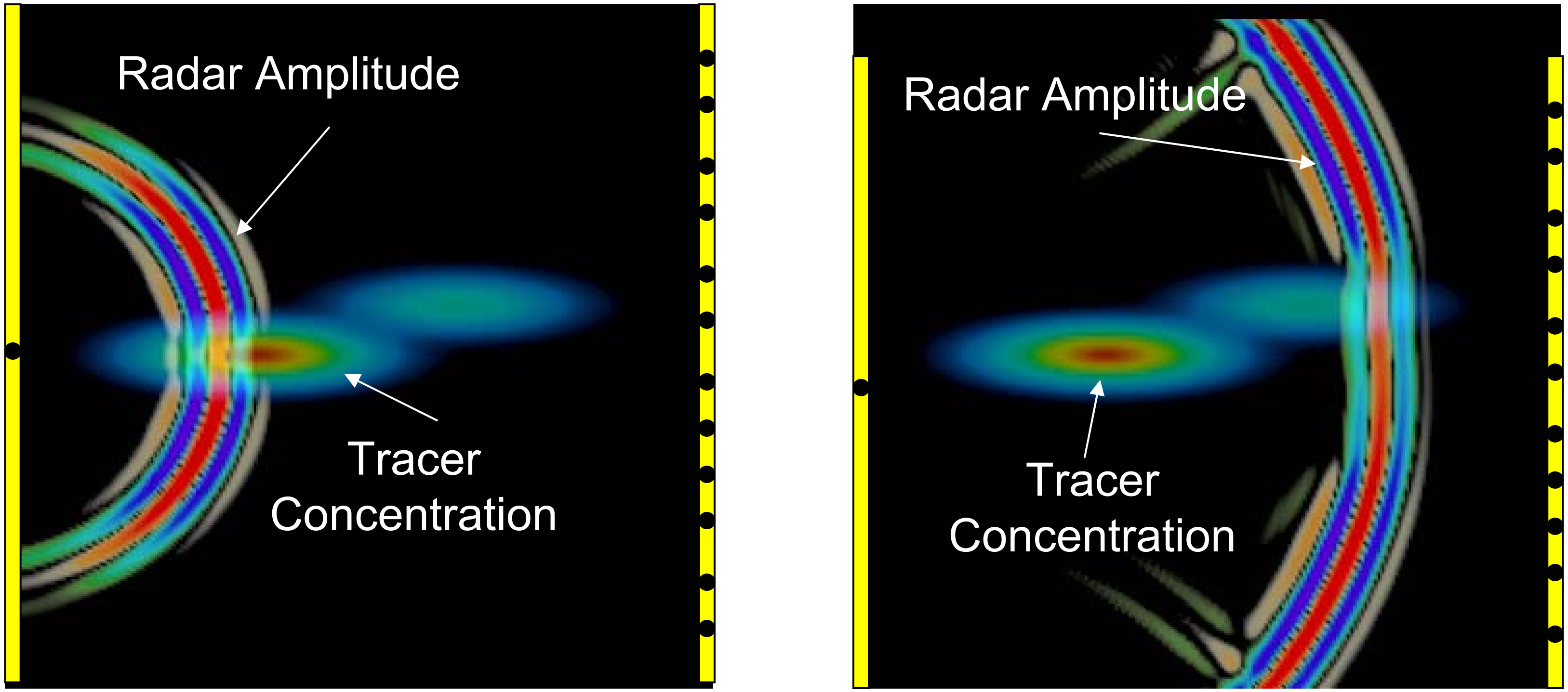
Nearby outcrop showing coarse, fluvial deposits believed to be analogous to those present at the BHRS.

Time Lapse Tracer Test



- Injection of electrically conductive tracer in well-B3
- Withdrawal in well-B6
- Radar data acquired in two panels: B1-B4, B2-B5
- Multi-level sampling in well A1 to capture the fluids passing through in different days.

Radar Attenuation Tomography



- Energy travelling through the tracer plume is attenuated.
- Time lapse change in amplitude is used to determine the tracer distribution.

Incorporating "better" Physics

- **Ray theory**

  Assumes waves propagate at infinite frequency

  Computationally fast & Requires less memory

- **Finite frequency wave propagation**

  Important advances in seismic problems (Woodward, 1992; Marquering, Dahlen, Nolet, 1999; Dahlen, Hung Nolet, 2000; Zhao, Jordan, Chapman, 2000; Hung, Dahlen, Nolet, 2001; Spetzler and Snieder, 2004; Nolet, 2005; de Hoop and van der Hilst, 2005 and many others)

  Important for resolving small scale features

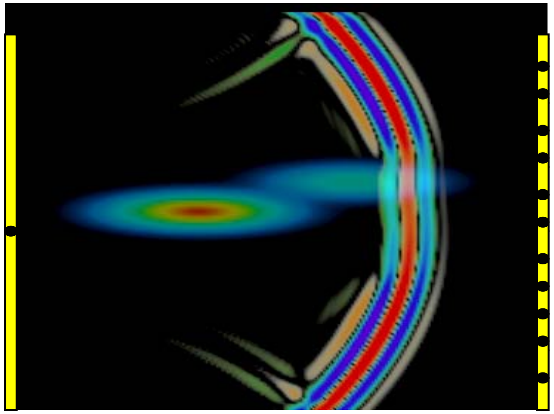
Why Finite Frequency is important?

- Wavelength ~ length scale of anomalies → scattering becomes important
- Ability to provide high-resolution image (less artifacts and less smearing)
- Reduced number of basis function required for model reconstruction
- Natural way to integrate data acquired at different frequencies.

Scattering Formulation

- Maxwell's Equation in 3D

$$\nabla \times E = -i\omega\mu H$$
$$\nabla \times H = (\sigma + i\omega\varepsilon) E + J_S$$



- Decompose the Electric Field into scalar and vector potential

$$E = -\nabla\phi - i\omega A$$

- Under high frequency approximation E ~ has contribution from vector potential (Wave regime)

$$E \approx -i\omega A$$

Scattering Formulation

- Finally we get the Helmholtz equation for electric field

$$\nabla^2 E + k^2 E = -i\omega\mu J_S(r_S) = S(\omega)\delta(r - r_S)$$

$$\nabla^2 E + \left(\omega^2 \mu\varepsilon - i\omega\mu\sigma\right)E = S(\omega)\delta(r - r_S)$$

$$(\varepsilon + \delta\varepsilon)$$

$$(\sigma + \delta\sigma)$$

Velocity tomography

Attenuation tomography

Scattering Formulation

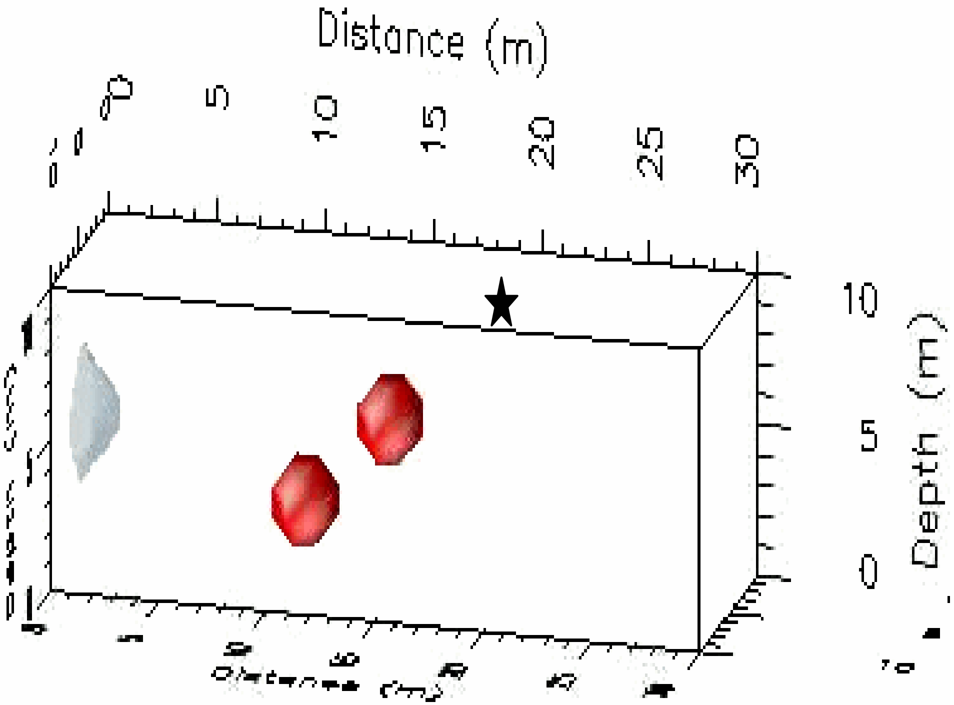
- Background field $$\nabla^2 E_0 + \frac{\omega^2}{c_0^2} E_0 = S(\omega)\delta(r - r_S)$$
- Solution for background field $$E_0(r) = G(r, r_S, \omega) S(\omega)$$
- Scattered field equation

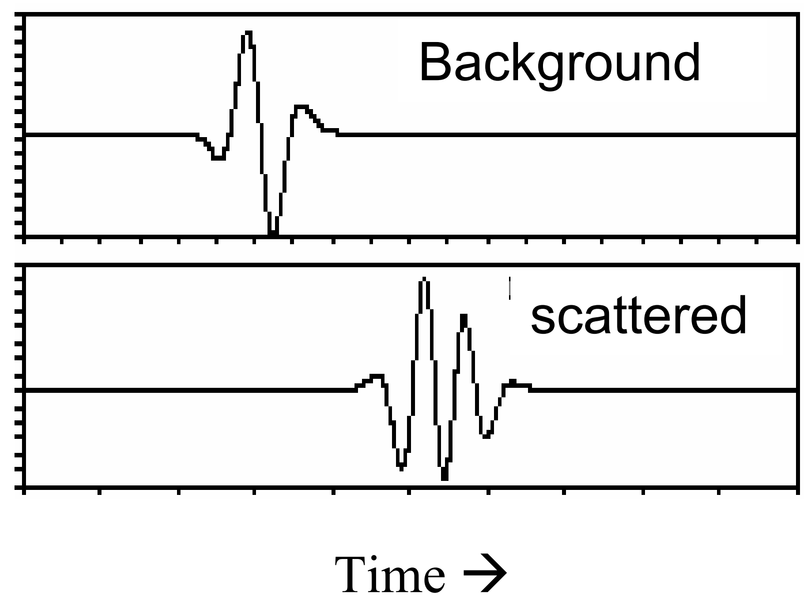
$$\nabla^2 \delta E + \frac{\omega^2}{c_0^2} \delta E = 2 \frac{\delta c\, \omega^2}{c_0^3} \left(E_0 + \delta E\right)$$

- Scattered field solution

$$\delta E(r, \omega) = 2 \frac{\omega^2}{c_0^3} \int G(r, r') \delta c(r') \left(E_0(r') + \delta E(r')\right) dr'$$

3D Forward Scattering Example





Routh, P. S., and Johnson, T. C., 2005, Multiple scattering in 3D georadar problem, SEG expanded abstracts, p1065-1068.

First Order Solution: Frechet Kernel

- Born solution for velocity perturbation:

$$\delta E(r,\omega) = 2\frac{\omega^2 S(\omega)}{c_0^3}\int G(r,r')G(r',r_S)\delta c(r')dr'$$

Scatter to Receiver — Source to Scatter

- Born solution for conductivity perturbation:

$$\delta E(r,\omega) = i\omega\mu\, S(\omega)\int G(r,r')G(r',r_S)\,\delta\sigma(r')\,dr'$$

$$\delta\sigma(r) = \sum_{k=1}^{M}\delta\sigma_k\,\Gamma_k(r)$$

Discrete Solution: Frechet kernel for full waveform inversion

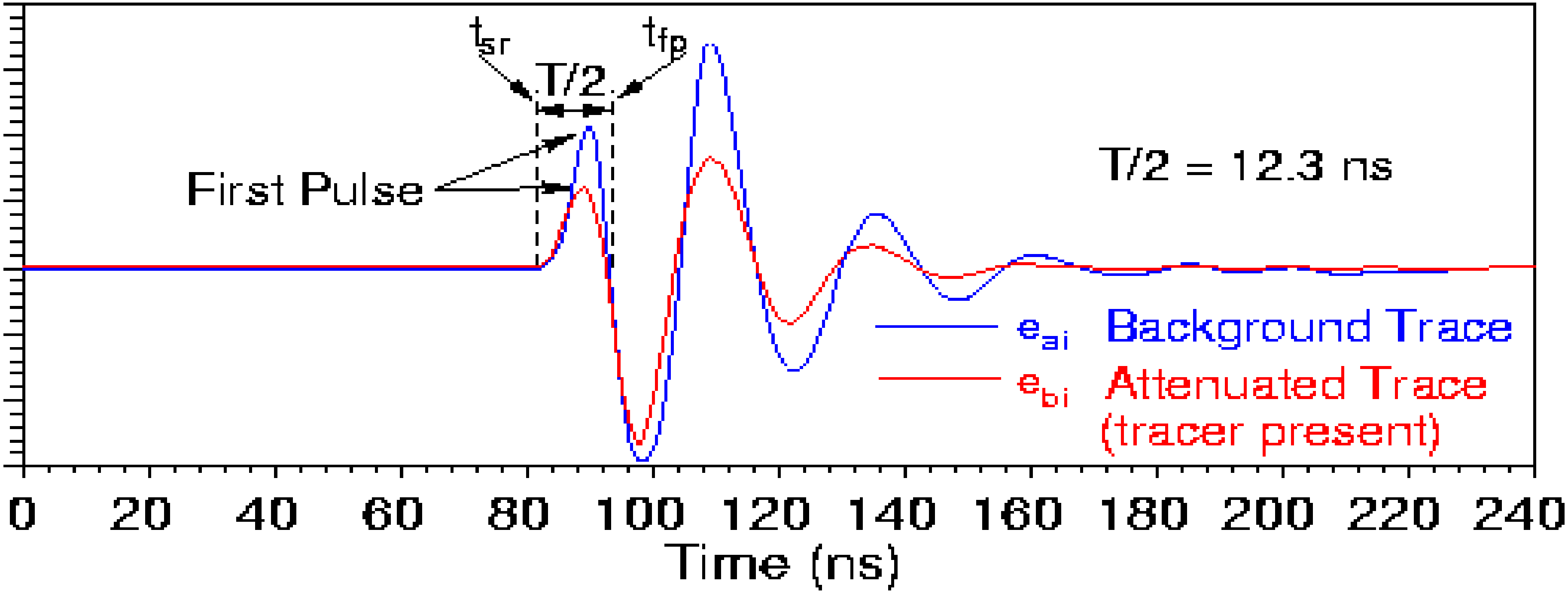
- velocity perturbation:

$$\delta E(r,\omega) = 2\frac{\omega^2 S(\omega) v_k}{c_0^3}\sum_{k=1}^{M} G(r,r_k)G(r_k,r_S)\delta c_k$$

- conductivity perturbation:

$$\delta E(r,\omega) = i\omega\mu\, S(\omega) v_k \sum_{k=1}^{M} G(r,r_k)G(r_k,r_S)\delta\sigma_k$$

Amplitude Kernel

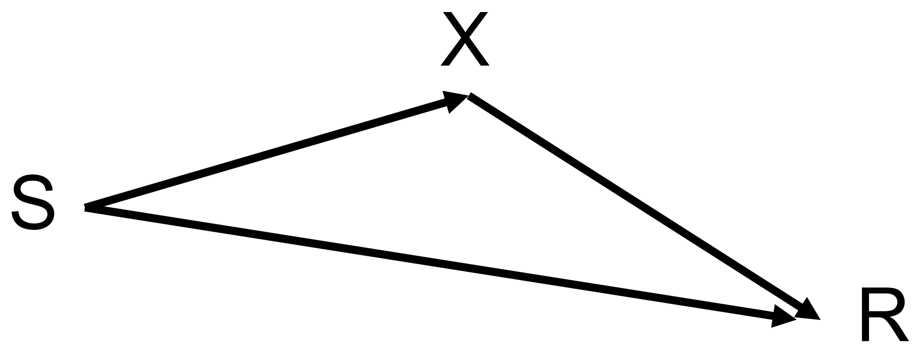


$$\ln\left(\frac{E_1}{E_o}\right) \approx \frac{\int e_0(t)\,\delta e(t)\,dt}{\int e_0^2(t)\,dt} = \operatorname{Re}\frac{\int_0^\infty E_0^*(\omega)\,\delta E(\omega)\,d\omega}{\int_0^\infty \left|E_0(\omega)\right|^2 d\omega}$$

Amplitude Kernel

$$\delta D(r) = \int K(r, r', \omega)\, \delta\sigma(r')\, dr'$$

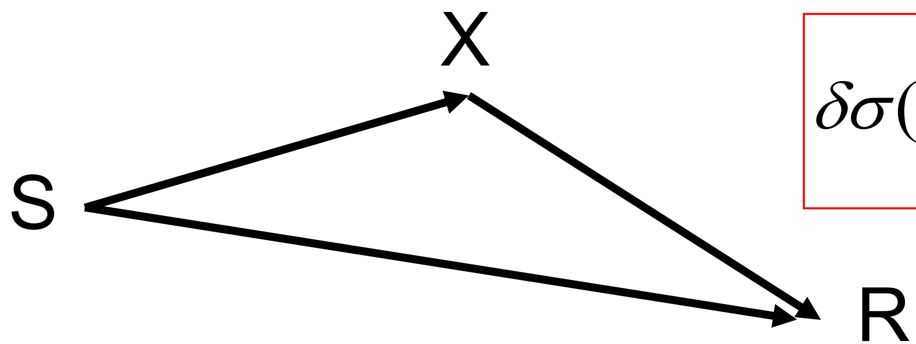
$$K(r, r', \omega) = \mathrm{Re}\, \frac{\int_0^\infty i\omega\mu\, |S(\omega)|^2\, G^*(r, r_S)\, G(r, r')\, G(r', r_S)\, d\omega}{\int_0^\infty |S(\omega)|^2\, |G(r, r_S)|^2\, d\omega}$$



Amplitude Kernel: Homogenous Green's Fn.

$$\delta D(r) = \int K(r, r', \omega)\, \delta\sigma(r')\, dr'$$

$$K(r,r',\omega) = -\frac{1}{4\pi}\frac{L_{SR}}{L_{SX}L_{XR}}\frac{\int_0^\infty \omega\mu |S(\omega)|^2 \mathrm{Sin}(\omega(t_{SX}+t_{XR}-t_{SR}))d\omega}{\int_0^\infty |S(\omega)|^2 d\omega}$$



$$\delta\sigma(r) = \sum_{k=1}^{M} \delta\sigma_k\, \Gamma_k(r)$$

2.5D Fresnel vs. full waveform sensitivities

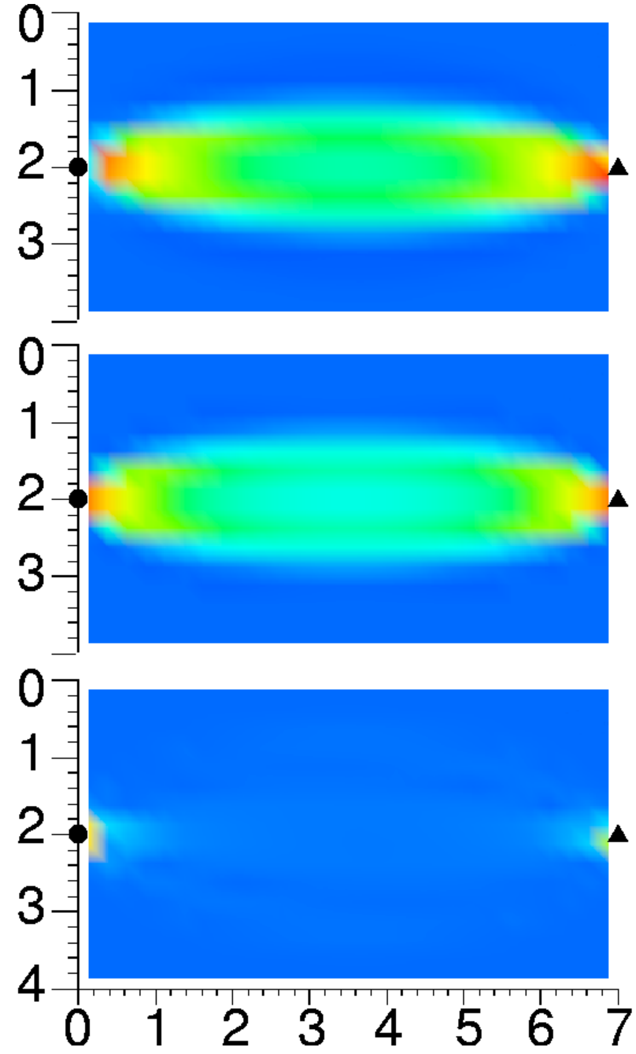
Exact Sensitivities

Comp. Time ~ 90 minutes

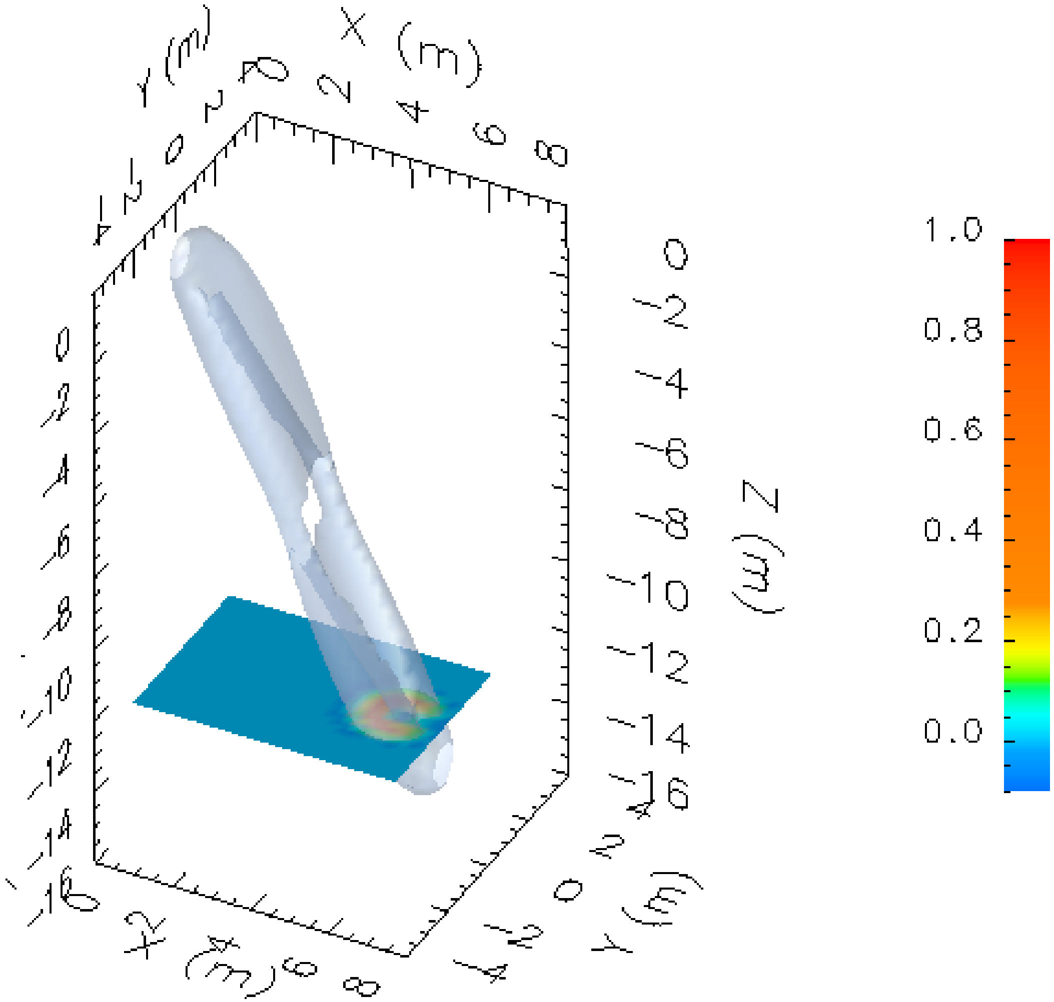
Scattering Sensitivities

Comp. Time < 2 second

Scattering Sensitivity Error

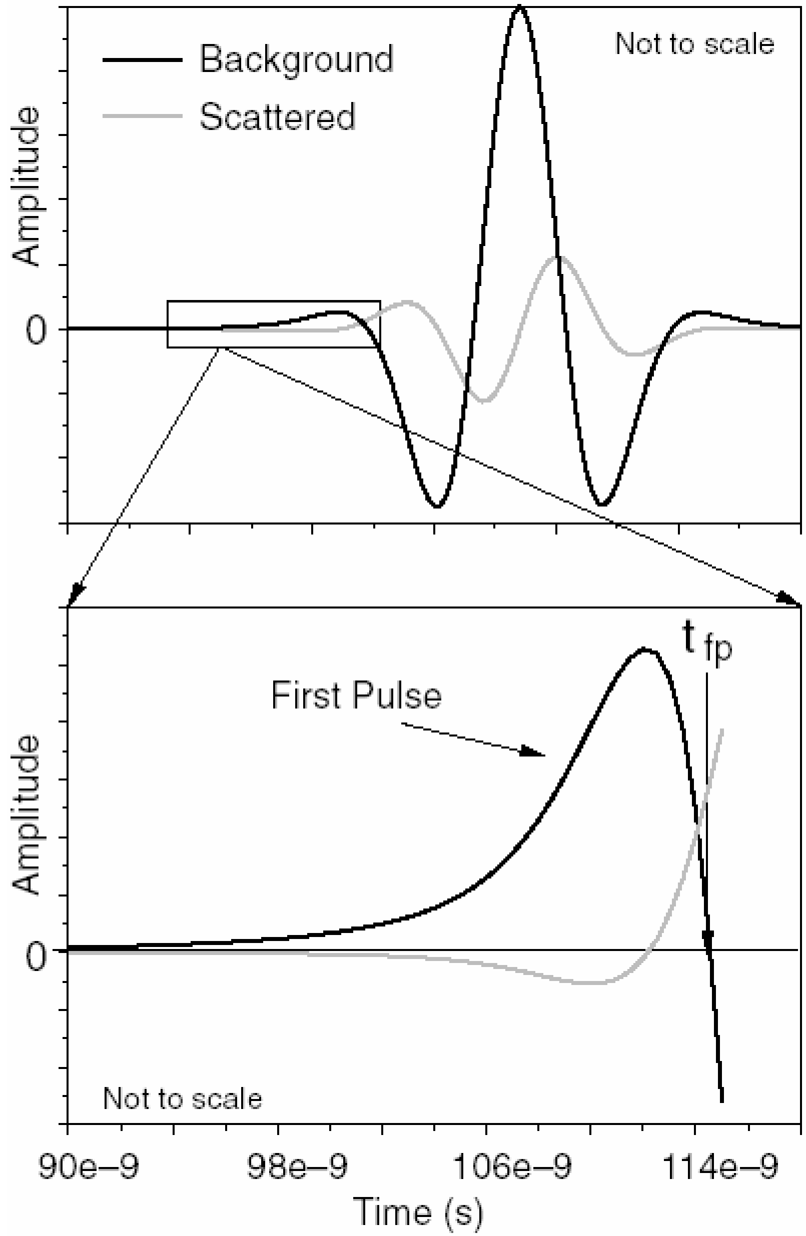


3D High angle Fresnel volume sensitivities



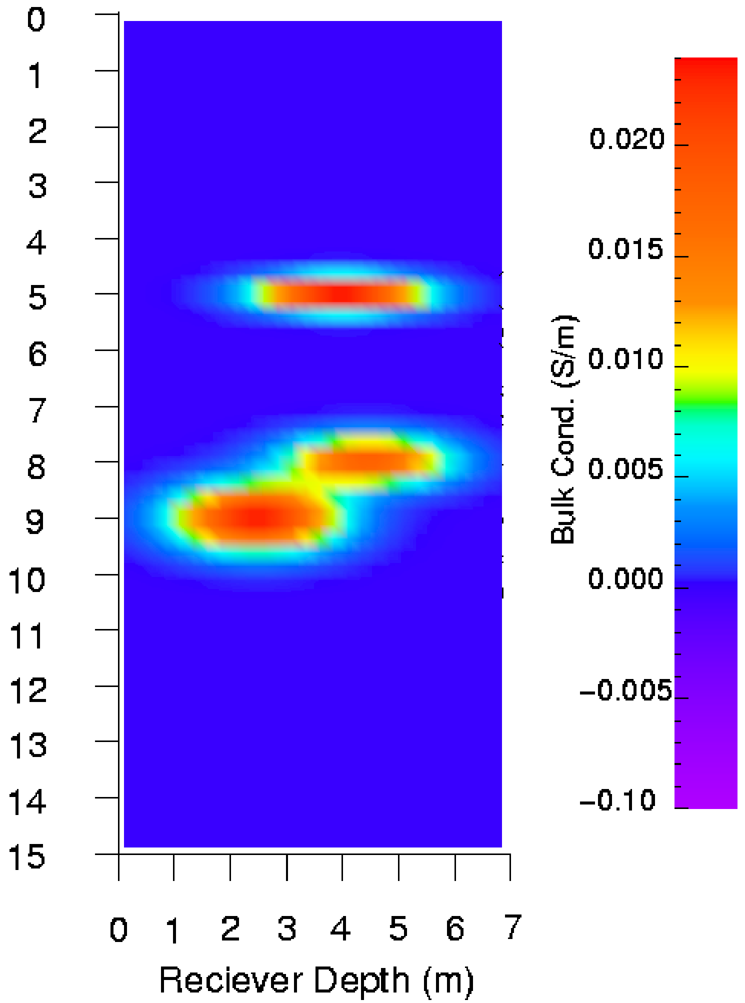
Johnson, T.,Routh, P. S., and Knoll, M. D., 2005, Fresnel volume georadar attenuation difference tomography, Geophysical Journal International, Vol. 162, p9-24.

Scatterer along the ray path

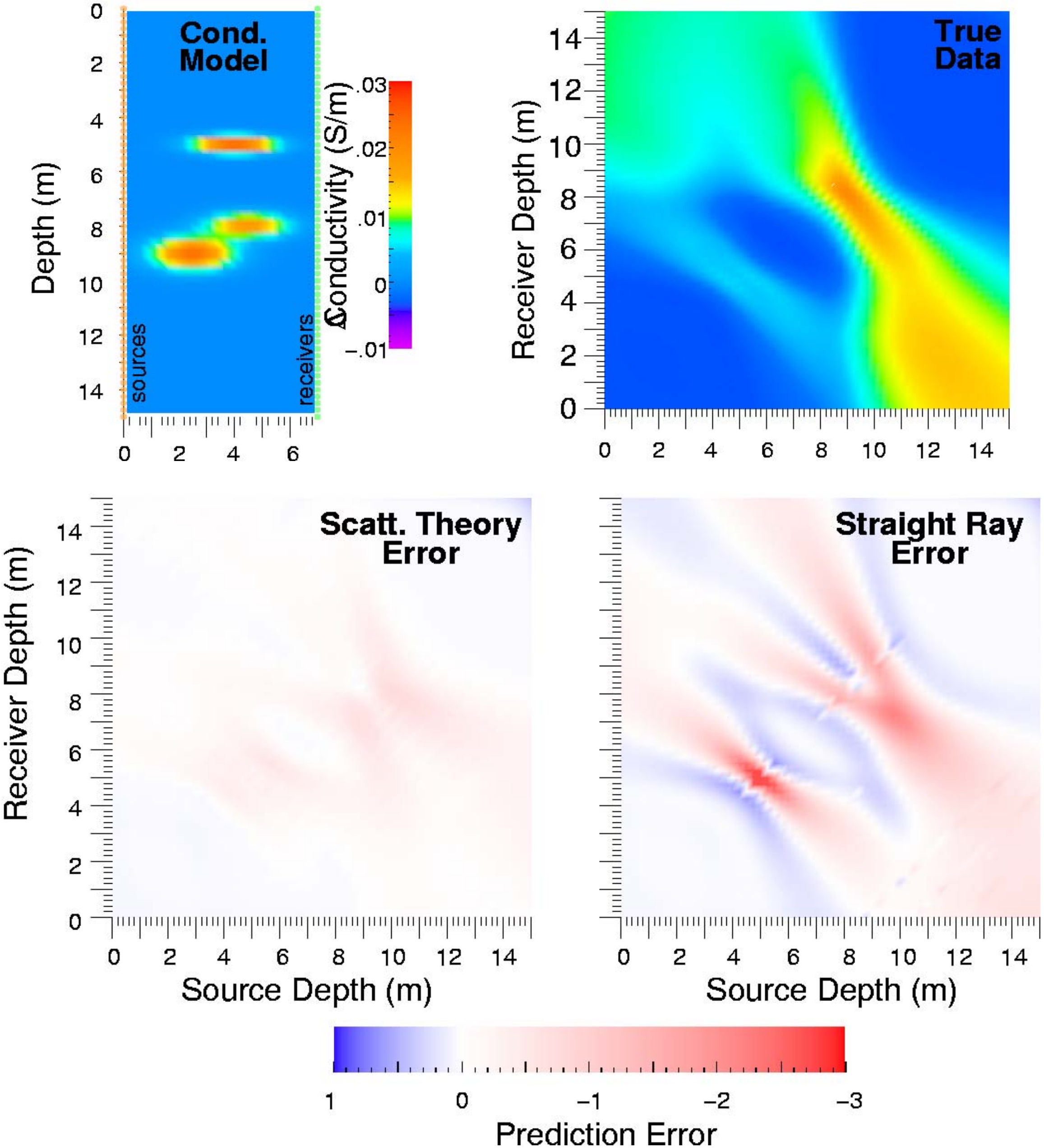


Synthetic Model

- Ray sensitivities ~ 5 sec.
- Scatt. theory sensitivities ~ 6 min.
- F.D.T.D (exact) sensitivities were computed on cluster and required 1.5 days on 100 processors

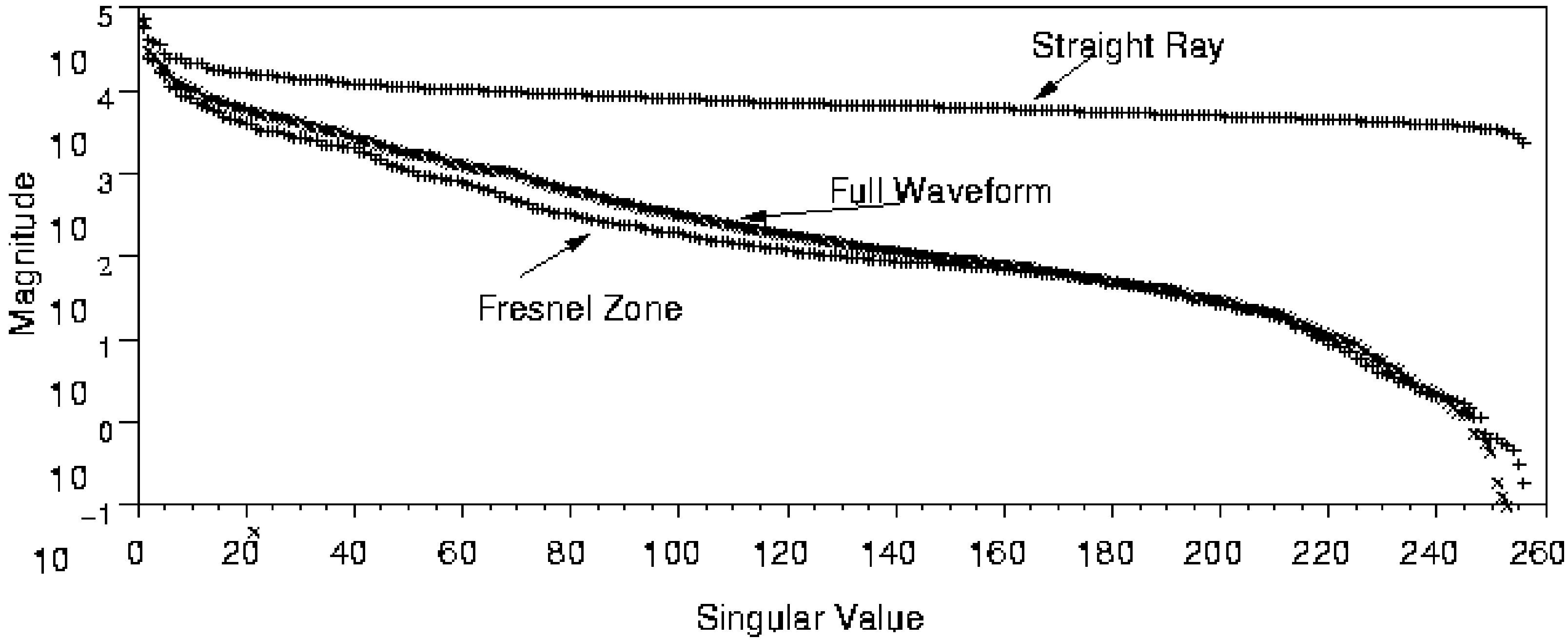


Forward Model Comparisons



• Data prediction from show fairly good match with full waveform computation

Singular value spectrum



$$\delta\sigma_{est} = \sum_{j=1}^{k} \left(\frac{U_j^T \delta D}{\Lambda_j} \right) V_j$$

Model basis functions

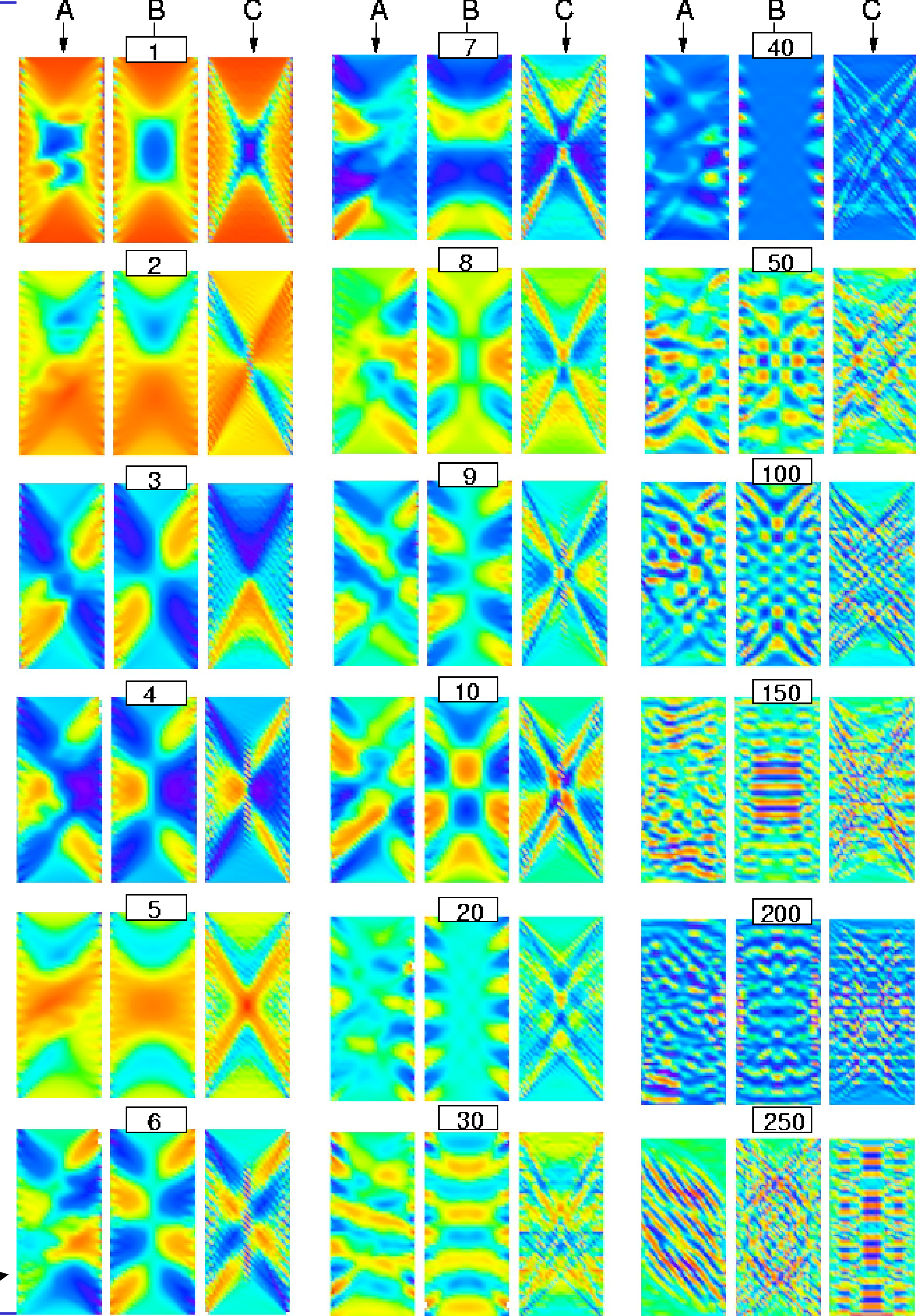
$$\delta\sigma_{est} = \sum_{j=1}^{k} \left(\frac{U_j^T \delta D}{\Lambda_j} \right) V_j$$

$$\delta\sigma_{est} = \sum_{j=1}^{k} c_j V_j$$

A – Full Waveform

B - Fresnel theory

C – Ray theory



Model basis functions

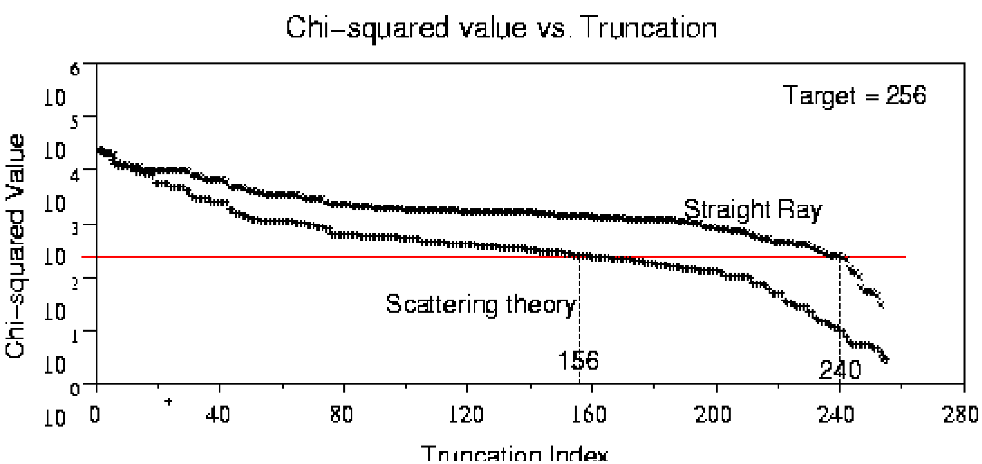
- Full waveform and Fresnel basis are similar:

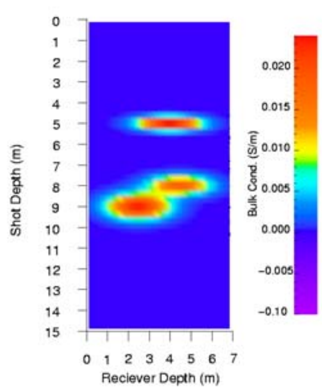
  - smooth localized structures
  - slowly varying with index

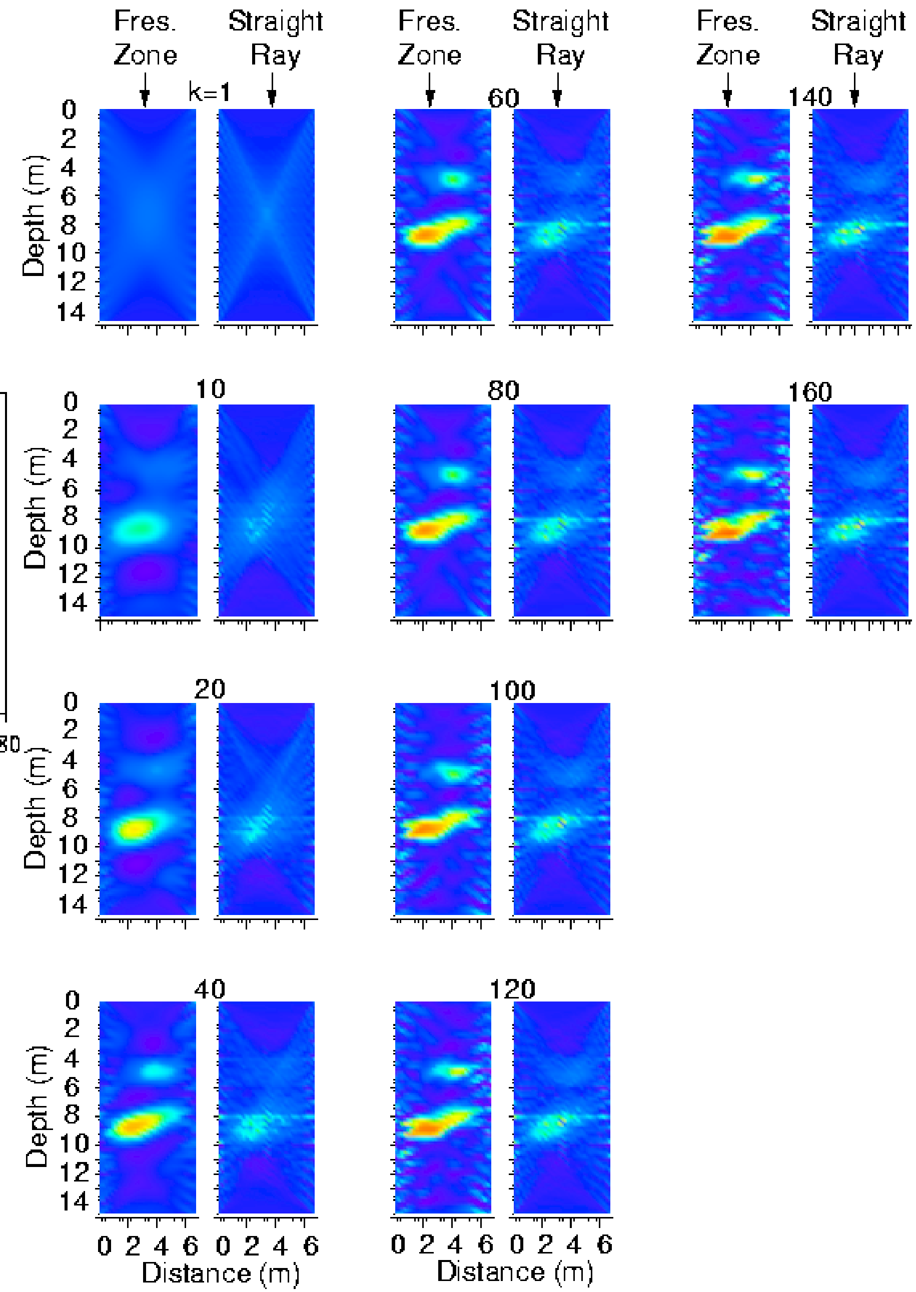
- Ray basis functions:

  - quickly become oscillatory
  - less localized
  - X-pattern dominates

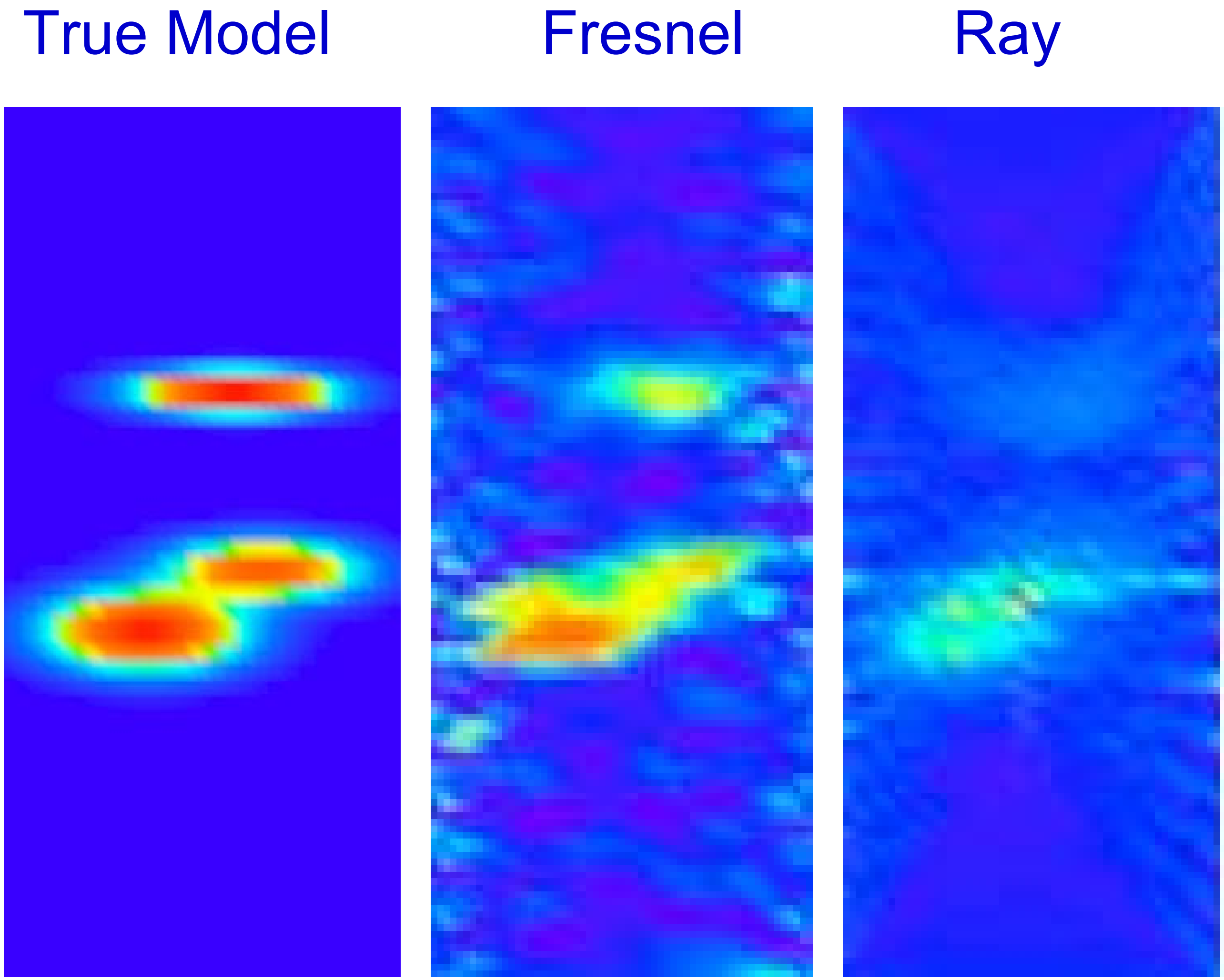
Tomography Results







Tomography Results



<u>Why finite frequency propagation is better?</u>

<u>Fresnel volume inversion</u>

- Fits data with **<u>fewer</u>** basis fn’s (reduced # of basis)
- Localizes peaks
- Fewer artifacts
- High resolution image (fine scale structures)

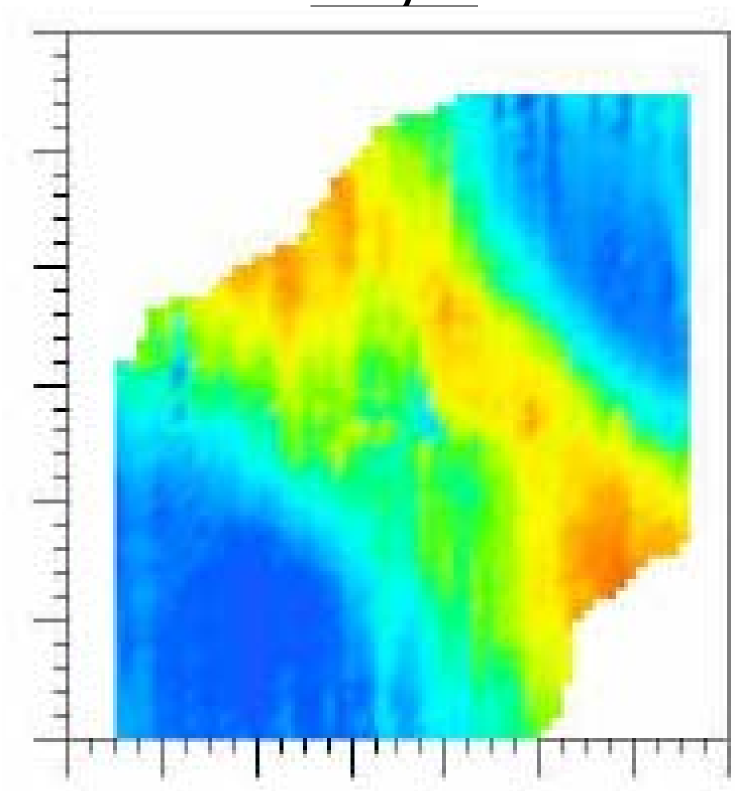
<u>Ray based inversion</u>

- Requires **<u>more</u>** high index oscillatory basis fn’s
- Unable to localize peaks (smear boundaries)
- Marked by X-pattern artifacts

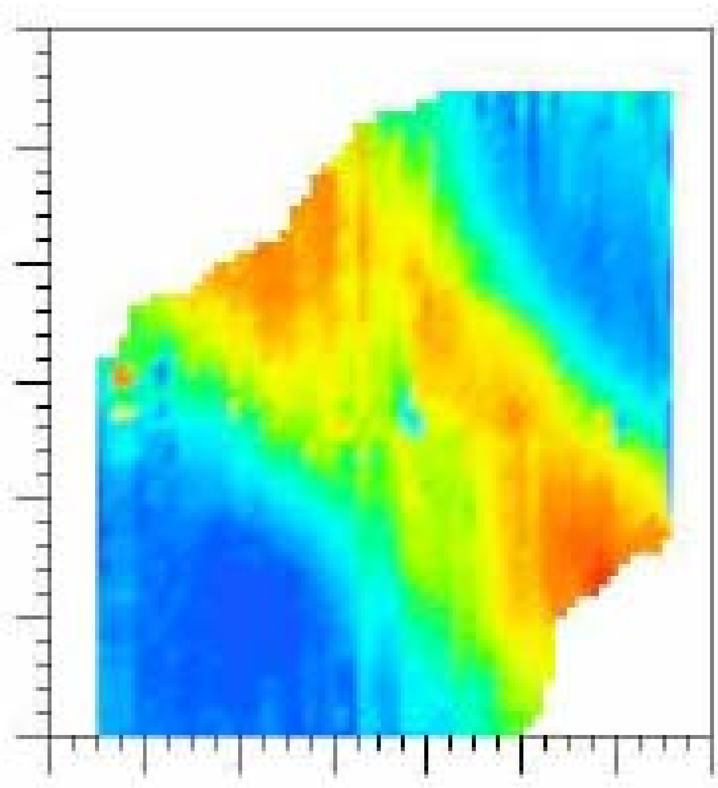
Field Tracer Time-Lapse Test: Data

Attenuation Data

Day 9



Day 10



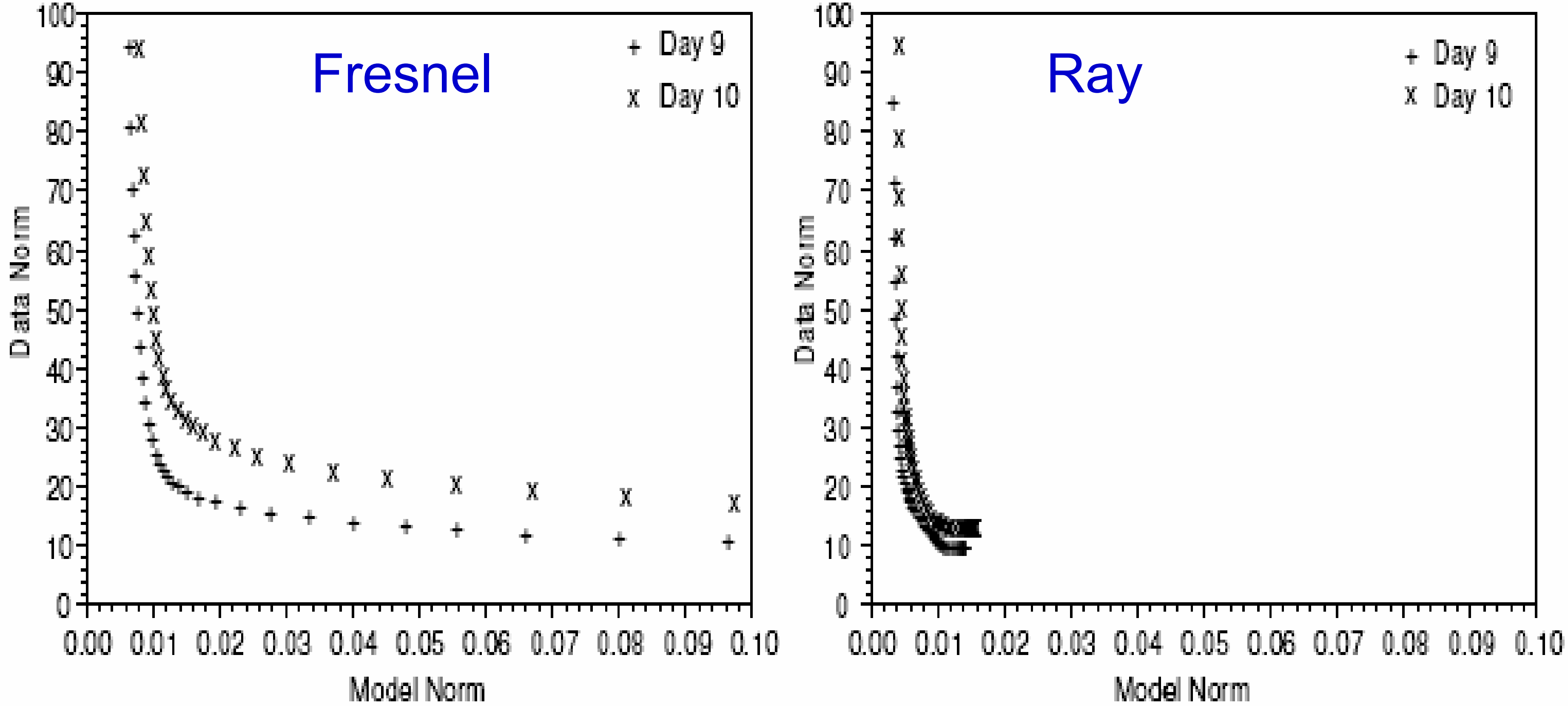
← RX (depth)

TX (depth) ⟶

TX (depth) ⟶

Field Tracer Time-Lapse Test: Regularization

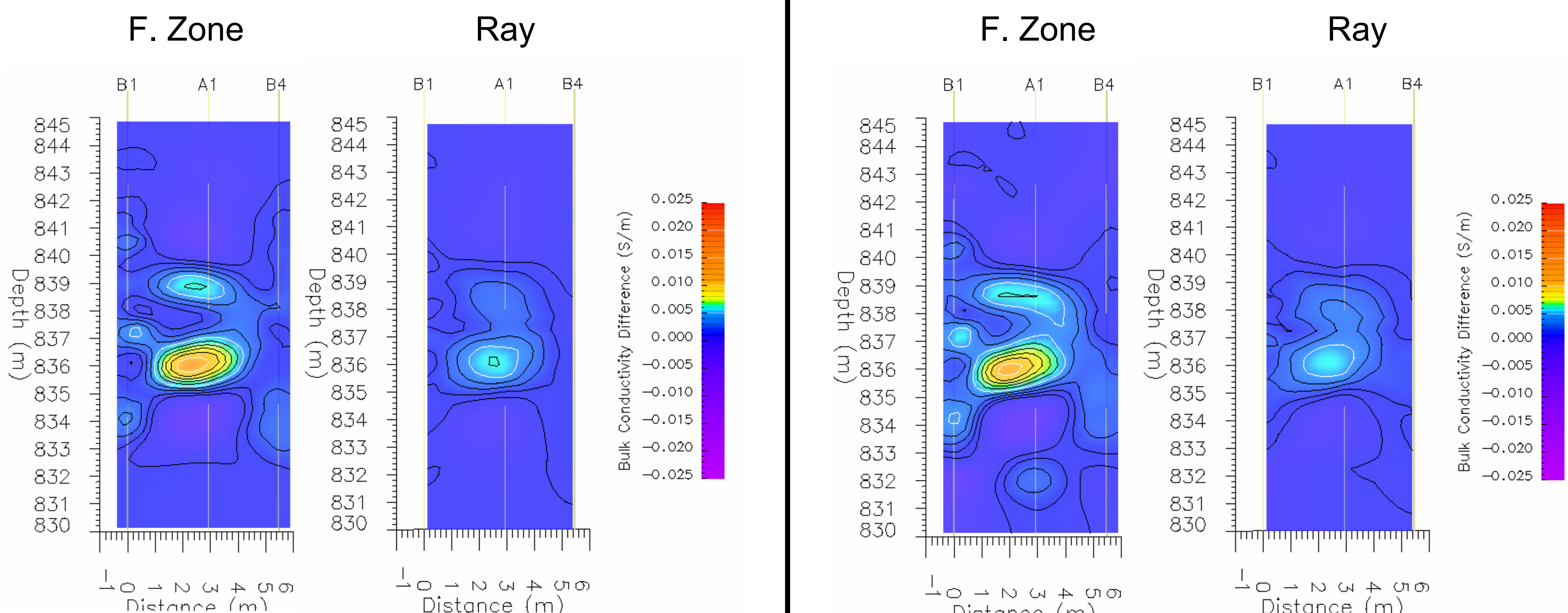
Regularize the time lapse inversion using L-Curve



Johnson, T.C., Routh, P. S., Barrash, W., and Knoll, M. D., 2006, Time lapse imaging of conductive tracer plume using fresnel zone GPR attenuation difference tomography, Geophysics (in review).

Field Tracer Time-Lapse Test: Inversion

Day 9 | **Day 10**



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Conclusions

- Fresnel volume tomography produces “better” images
- Has less artifacts compared to ray based inversion
- Requires less regularization
- Data fit is better
- Ability to integrate data at different frequencies
- Easier to handle data as a part of processing
- Marginal increase in computation and storage cost
- Future development: Time lapse inversion in 3D

Acknowledgements

- Environmental Protection Agency grant X970085-01-0
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