3D wave-equation prediction of multiples

DMITRI LOKSHTANOV
Norsk Hydro Research Centre, PO Box 7190, 5020 Bergen, Norway

Summary

We present two approaches for 3D wave-equation prediction. One is for generally irregular 3D sea-floor and arbitrary 3D structure below it. The second is a much faster scheme for locally 1D sea-floor with an arbitrary 3D structure below it. The latter is suitable for the majority of data from the North Sea. In both approaches the prediction and adaptive subtraction of multiples are performed in the same domain, therefore no additional sorting or additional transformations are required. All source-side and receiver-side multiples of all orders are suppressed simultaneously in one consistent step.

Introduction.

As long as successful migration / inversion with multiples remains a dream rather than a reality (especially migration / inversion without a proper initial velocity model), suppression of free-surface multiples remains the main step in processing offshore data. For marine data from areas with hard and/or irregular sea-floor, water-layer multiples and peg-legs are often the most troublesome part of the free-surface multiples. For such data the so-called wave-equation (WE) approaches (introduced by Berryhill and Kim, 1986 and by Wiggins, 1988) are powerful alternatives to the popular SRME method (Berkhout and Verschuur, 1997). Each iteration of SRME gives the sum of multiples of different orders, while the amplitude correction for predicted multiples of different orders should be different. The difference in the required correction for interfering multiples of different orders creates problems for adaptive subtraction, and this inconsistency between prediction and subtraction is a fundamental drawback of iterative SRME even for a 1D Earth. In our version of the WE approach all predicted multiples are split into three terms, where each term contains multiples which require the same amplitude correction. These three terms are prediction from the source-side, prediction from the receiver-side and prediction from the source-side after prediction from the receiver-side (a second-order term in the deconvolution). Independently of water-layer depth all water-layer multiples and peg-legs of all orders are suppressed simultaneously by adaptive subtraction of these three terms in one or a few time windows. The great advantage of SRME is that it does not require any structural information and its potential ability to predict a larger class of multiples than just water-layer multiples and peg-legs. At the same time when data themselves are used as a prediction operator, then obviously noise in data and poor sampling significantly degrade the prediction quality. This is specially the case for 3D prediction with current quasi-3D marine acquisition. SRME requires the same dense sampling between sources as between receivers. Therefore it is hardly possible to talk about 'true' 3D SRME when the source interval in the crossline direction (swath distance) is several hundred metres. Of course, one can try to reconstruct the input data (Levin, 2002; Hokstad, 2004; Pica et al., 2005) or to use inversion during prediction (van Dedem and Verschuur, 2001). However with such poor sampling of input data these approaches can hardly work for data from areas with strong lateral variations. The requirements for data sampling for 3D WE approaches are less severe than for 3D SRME. Indeed, with current marine acquisition (dense sampling between receivers for each shot) we can accurately perform 3D data extrapolation through the water-layer from the receiver-side, and this leads to accurate prediction of all 'pure' water-layer multiples (with multiple diffractions included) and of all receiver-side peg-legs. The 3D WE extrapolation from the source-side requires additional assumptions or better sampling between shots in the crossline direction. Therefore, 'true' 3D prediction of source-side peg-legs cannot be performed with current marine acquisition. It is important to underline that, in contrast to 3D SRME, the 3D WE approach has problems with 'true' 3D prediction not for all multiples, but only for some of them. Finally, as in WE migration, we can use very different extrapolation operators for data from different areas: fully 2D/3D extrapolation for a generally arbitrary 2D/3D Earth, or much faster approaches based on the assumption of 'locally' 1D sea-floor with arbitrary 2D/3D structure below it. Note that the latter is the case for the majority of data from the North Sea.

Multiple suppression operator

The exact operator for removal of water-layer multiples and peg-legs from the 2D or 3D input data D (in whatever domain) has the following form (Lokshtanov, 2000):

$$F = (I + P_s)(I + P_g)D - P_s D_w, \tag{1}$$

where I is an identity operator; P_g and P_s are the ideal operators for forward extrapolation of input data through the water layer (down and up and including correct reflectivity from the sea-floor) from the receiver side and source side respectively; D_w is the primary reflection from the water-bottom and F is the resulting multiple free data. In reality we do not know the correct reflectivity from the sea-floor. Therefore we apply (1) in an adaptive manner. First we do kinematic prediction of multiples using the known geometry of the sea-floor and neglecting reflectivity effects (see next sections). Then we apply trace by trace adaptive subtraction of the predicted multiples. Both prediction and subtraction are performed using the Radon transformed CMP or CS gathers. For each *p* trace the adaptive subtraction operator has the form:

$$f(\tau) = d(\tau) + r_g(\tau) * d_g(\tau) + r_s(\tau) * d_s(\tau) + r_{sg}(\tau) * d_{sg}(\tau), \qquad (2)$$

where $d(\tau)$, $d_g(\tau)$, $d_s(\tau)$, $d_{sg}(\tau)$ are *p*-traces for the input data and the results of extrapolation from the receiver side, source side (of muted input data) and source side after receiver side respectively. The filters $r_g(\tau)$, $r_s(\tau)$, $r_{sg}(\tau)$ account for angle-dependent reflection coefficients from the water bottom and small phase shifts due to imperfect knowledge of the water-bottom geometry. The filters are estimated from the criterion of minimum energy of *f*. According to (1)-(2) all water-layer multiples and peg-legs of all orders are suppressed simultaneously in one consistent step (in one or a few time windows).

Fast 3D wave-equation prediction of multiples

Here we assume a locally 1D sea-floor with arbitrary 3D structure below it. Note that this is the case for the majority of data from the North Sea, for example the data from Oseberg, Troll, Brage, Grane, Njord and Farsund areas. The general case of irregular sea-floor is considered in the next section.

For a locally 1D sea-floor it might seem that the fastest and simplest prediction operator is the phase-shift operator. With the phase shift operator the prediction from the receiver-side must be performed on the Radon (or Fourier) transformed common shot gathers, while the prediction from the source-side must use the Radon (Fourier) transformed common receiver gathers (with all the sampling issues involved). One must then bring all predicted multiples into one common domain for adaptive subtraction. In our fast scheme (Lokshtanov, 2005) all these operations are performed on tau-p transformed CMP gathers (but not with simple phase-shift operator), therefore no additional transformations or sorting are required.

Denote input data by $D(p, x, y, \omega)$ - the result of linear Radon transform (with respect to inline offset) of the CMP gather with inline coordinate x and receiver crossline coordinate y; p is the ray parameter of the transform and ω is the temporal frequency. We assume that the input data *D* belong to the shots with crossline coordinate equal to zero (the case of flip flop shooting will be discussed later), therefore we consider the data from the same swath. With these notations the result of receiver-side prediction $D_r(p, \tilde{x}, \tilde{y})$ (Radon transformed CMP gather with CMP coordinates $(\tilde{x},\ \tilde{y}/2)$ can be obtained as:

$$D_r(p, \tilde{x}, \tilde{y}) = \frac{\omega^2}{(2\pi)^2} \iint D(p, x, y) \left[\iint \exp\left\{ i\omega \left(p_d\ (\tilde{x} - x) + p_{yr}(\tilde{y} - y) + 2q_r z \right) \right\} dp_d\ dp_{yr} \right] dx\, dy, \qquad (3)$$

$$\text{where}\quad q_r = \left(\frac{1}{c^2} - p_r^2 \right)^{1/2}, \quad p_r = \left(p_{xr}^2 + p_{yr}^2 \right)^{1/2}, \quad p_{xr} = p - \frac{p_d}{2}\ , \qquad (4)$$

z is local sea-floor depth and c is water velocity. For given $\tilde{x}, \tilde{y}, x, y$ the inner integral in the square brackets is calculated analytically by the stationary phase approximation: $\varphi = p_d\ (\tilde{x} - x) + p_{yr}(\tilde{y} - y) + 2q_r z$,

$$\frac{\partial\varphi}{\partial p_d} = 0 \ \Rightarrow\ (x - \tilde{x}) = z\, \mathrm{tg}\alpha_r \cos\beta_r\,, \quad \frac{\partial\varphi}{\partial p_{yr}} = 0 \ \Rightarrow\ (\tilde{y} - y) = z\, \mathrm{tg}\alpha_r \sin\beta_r\,, \qquad (5)$$

where α_r, β_r are receiver-side vertical angle and azimuth respectively, $\mathrm{tg}\alpha_r = p_r / q_r$, $\cos\beta_r = p_{xr} / p_r$. Formulas (3)-(5) are a 3D extension of 2D results by Lokshtanov (2000). According to (3)-(5) each *p* trace of the CMP gather after prediction is simply obtained as a sum of time-delayed input traces with the same *p*. The additional factor $i \cdot \omega$ (which appears from ω^2 in (3) and i/ω from the stationary phase approximation) can then be accounted for in the frequency domain or simply neglected, assuming that adaptive subtraction filters will automatically include it.

With poor sampling of shots in the crossline direction, a 3D prediction of multiples $D_s(p,\tilde{x},\tilde{y})$ from the source-side cannot be obtained without additional assumptions. One possible assumption is that $p_{ys} = p_{yr}$, where p_{ys}, p_{yr} are the crossline slownesses from the source-side and receiver-side respectively. With this assumption an approximate 3D prediction from the source side can be obtained similar to (3)-(5):

$$D_s(p,\tilde{x},\tilde{y}) \approx \frac{\omega^2}{(2\pi)^2}\iint D(p,x,y)\left[\iint \exp\left\{i\omega\left(p_d\,(\tilde{x}-x) + p_{yr}\,(\tilde{y}-y) + 2q_s z\right)\right\} dp_d\, dp_{yr}\right] dx\, dy\,, \quad (6)$$

$$\text{where } q_s = \left(\frac{1}{c^2} - p_s^2\right)^{1/2}, \quad p_s = \left(p_{xs}^2 + p_{ys}^2\right)^{1/2}, \quad p_{xs} = p + p_d/2 \text{ and } p_{ys} = p_{yr}\,. \quad (7)$$

As in the previous case for receiver-side prediction, the inner double integral in the square brackets is calculated analytically by the stationary phase approximation: $\varphi = p_d\,(\tilde{x}-x) + p_{yr}\,(\tilde{y}-y) + 2q_s z$,

$$\frac{\partial\varphi}{\partial p_d} = 0 \;\Rightarrow\; (\tilde{x}-x) = z\,\mathrm{tg}\alpha_s \cos\beta_s\,, \quad \frac{\partial\varphi}{\partial p_{yr}} = 0 \;\Rightarrow\; (y-\tilde{y}) = z\,\mathrm{tg}\alpha_s \sin\beta_s\,, \quad (8)$$

where α_s, β_s are source-side vertical angle and azimuth respectively, $\mathrm{tg}\alpha_s = p_s/q_s$, $\cos\beta_s = p_{xs}/p_s$.

The assumption $p_{ys} = p_{yr}$ also gives an immediate formalism to mix data with flip flop shooting. The formulas (3)-(8) remain the same, where $y, \tilde{y}$ now stand for the difference between the crossline coordinate of the receiver and the corresponding source for input data and results of prediction respectively.

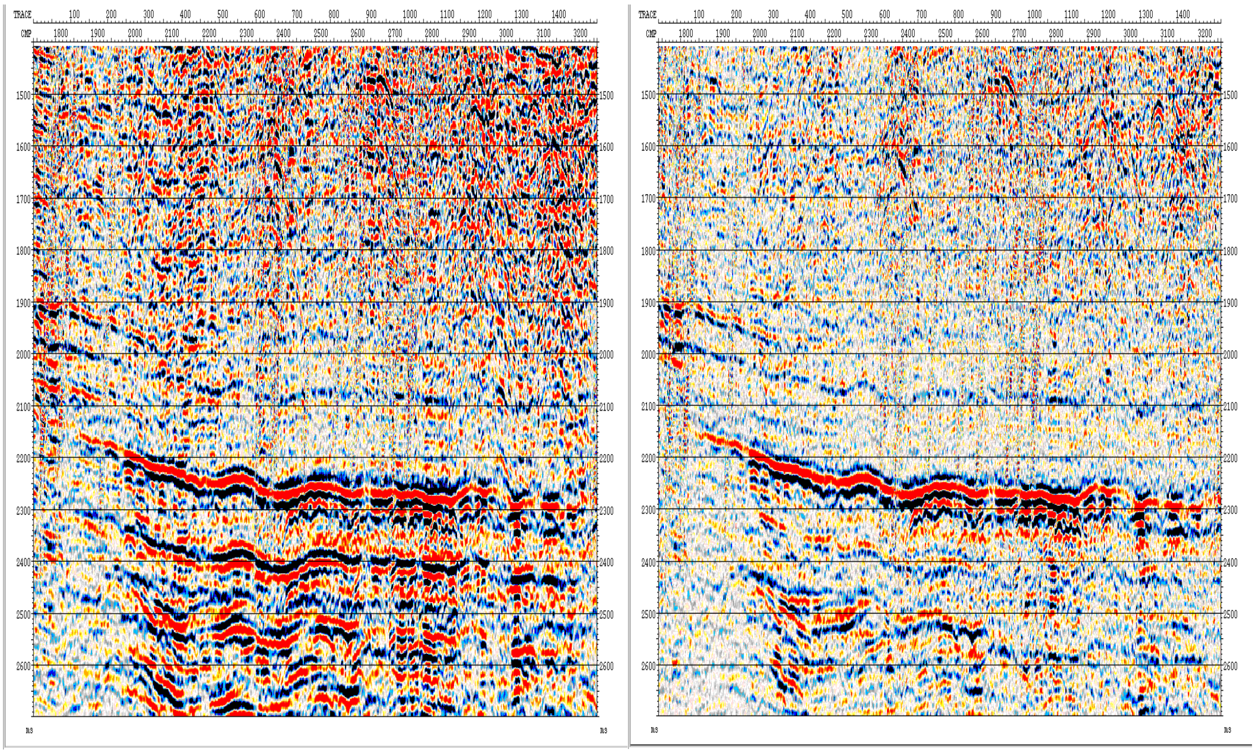

Figure 1. Constant *P* section before (left) and after 3D prediction / subtraction of multiples (right)

3D prediction of multiples from an irregular sea-floor

For receiver-side prediction of multiples from a 3D irregular sea-floor, we use a 3D extension of a 2D approach by Lokshtanov, 2004. The procedure is performed shot by shot using the Radon transformed common shot gathers. Denote by $D(p_x, y_r, \omega)$ the common shot (CS) gathers Radon transformed with respect to inline offset, where p_x is the receiver-side horizontal ray parameter; y_r is the crossline coordinate of the streamer and ω is the temporal frequency. We assume that inline and crossline coordinates of the source are equal to zero.

For each frequency the recorded CS wavefield is decomposed into plane waves and then each plane wave is extrapolated down to the sea-floor:

$$W(x,y,z(x,y)) = \frac{\omega^2}{4\pi^2}\iint D(p_x,y_r)\exp\{i\omega p_x x\}\left[\int \exp\{i\omega[p_y(y-y_r)+q_r z(x,y)]\}dp_y\right]dp_x dy_r, \quad (9)$$

where $z(x,y)$ is the depth of the water-bottom at lateral position x,y; $q_r = (1/c^2 - p_x^2 - p_y^2)^{1/2}$ is the vertical slowness; c is water velocity. In the prediction procedure we assume that the reflection coefficients are equal to one for all angles of incidence and all reflection points along the sea-floor. Therefore the result (9) defines the reflected wavefield along the water-bottom. This wavefield is constructed by superposition of phase shifted recorded plane waves, therefore the effects of multiscattering along the sea-floor are not accounted for. The inner integral in the square brackets is calculated analytically by the stationary phase approximation. At the stationary point $\varphi = p_y(y-y_r) + q_r z = \tilde{q}_r\left[z^2 + (y-y_r)^2\right]^{1/2}$, where $\tilde{q}_r = (1/c^2 - p_x^2)^{1/2}$. The next step is forward extrapolation of the reflected wavefield from the sea-floor up to the free-surface (Wenzel et al., 1990) and then decomposition again of the reflected/scattered wavefield into plane wave contributions. The Radon transformed CS gathers after prediction $D_g(p_x, y_r)$ are obtained as:

$$D_g(p_x,y_r) = \frac{\omega}{2\pi}\iint W(x,y)\exp\{-i\omega p_x x\}\left[\int C\cdot\exp\{i\omega[p_y(y_r-y)+q_r z(x,y)]\}dp_y\right]dx\,dy. \quad (10)$$

As in (9), the inner integral in (10) is calculated analytically by the stationary phase approximation. The factor *C* depends on the local slope of the boundary and is also a slowly varying function of p_x and p_y. Since we do not account for reflectivity during kinematic prediction, this factor can be dropped.

The combination of strongly irregular sea floor and current acquisition with poor sampling between shots in the crossline direction does not allow true 3D prediction from the source side. For source-side prediction we use a 2D approach by Lokshtanov (2004). The source-side prediction is also performed from the Radon transformed CS gathers and results in the Radon transformed CS gathers. Therefore, as above, all predictions and adaptive subtraction are performed in the same domain.

Conclusions

When the main free-surface multiples are water-layer multiples and peg-legs, the WE approach is a powerful alternative to the SRME method, both in terms of the quality of the results and computational time. In our version of the WE method all water-layer multiples and peg-legs are suppressed simultaneously in one consistent step. The 3D WE approach has fewer sampling problems than 3D SRME. Current quasi 3D marine acquisition permits accurate 3D prediction of 'pure' water-layer multiples and receiver-side peg-legs. 3D prediction from the source side requires additional assumptions.

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