Stabilizing wavefield extrapolation with locally WKBJ operator symbols

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1. Introduction
2. Standard stabilization techniques
3. An easier way to do it?
4. The lwkbj operator
5. Images and results
6. Conclusions

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- The stabilization process is complicated.
- Can we find a simple method?

Generalized PSPI

Infinitesimal extrapolator

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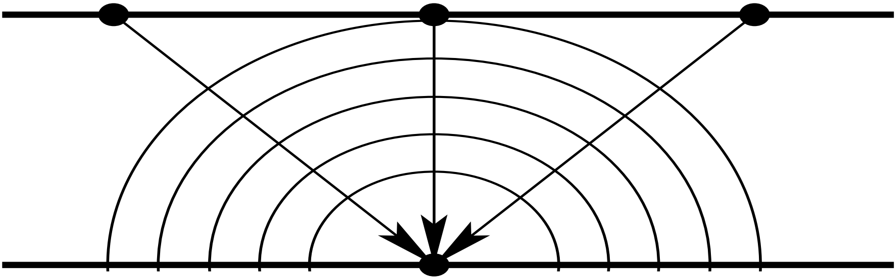
$$\Psi(x, z+\Delta z, \omega) = \mathbf{T}_{\alpha(z:z+\Delta z)}\Psi(x,z,\omega) \approx \int_{\mathbb{R}} \phi(\xi, z, \omega)\alpha(x, \xi, \omega, z : z+\Delta z)e^{i\xi x}d\xi$$

where

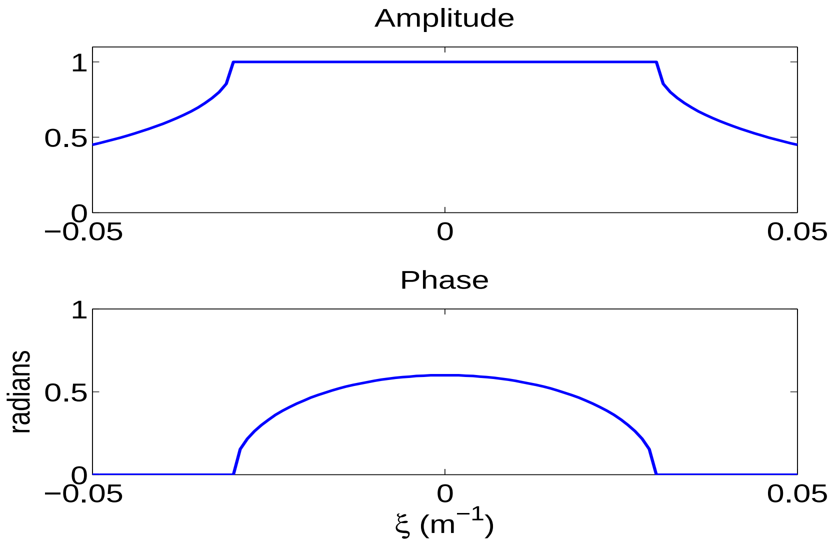
$$\alpha\left(x, \xi, \omega, z : z+\Delta z\right) = \begin{cases} \exp\left(i\Delta z\sqrt{\frac{\omega^2}{v(x)^2} - \xi^2}\right), & |\xi| \le \frac{|\omega|}{v(x)} \\ \exp\left(-\left|\Delta z\sqrt{\frac{\omega^2}{v(x)^2} - \xi^2}\right|\right), & |\xi| > \frac{|\omega|}{v(x)} \end{cases}$$

GPSPI CONCEPTUALLY

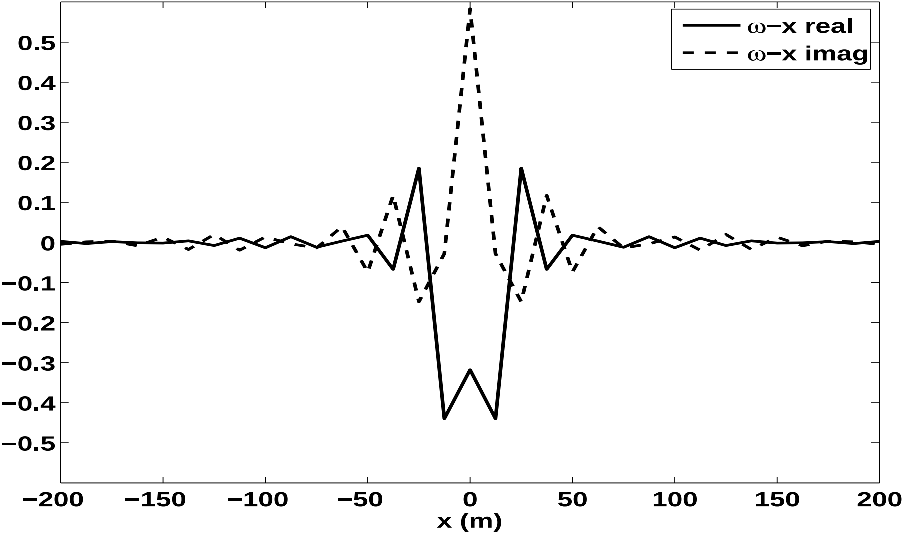
The locally homogeneous symbol



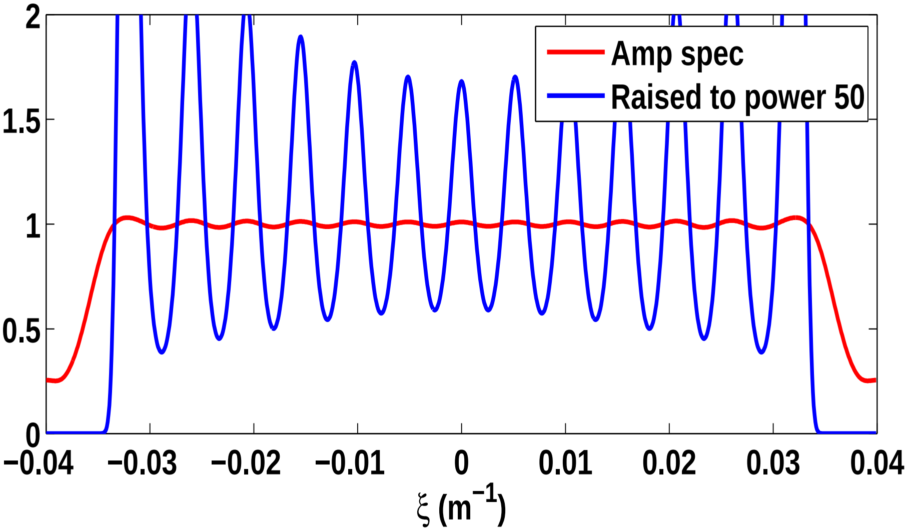
Wavefield extrapolation: the symbol α



Convolution equivalent



Truncation → Gibbs



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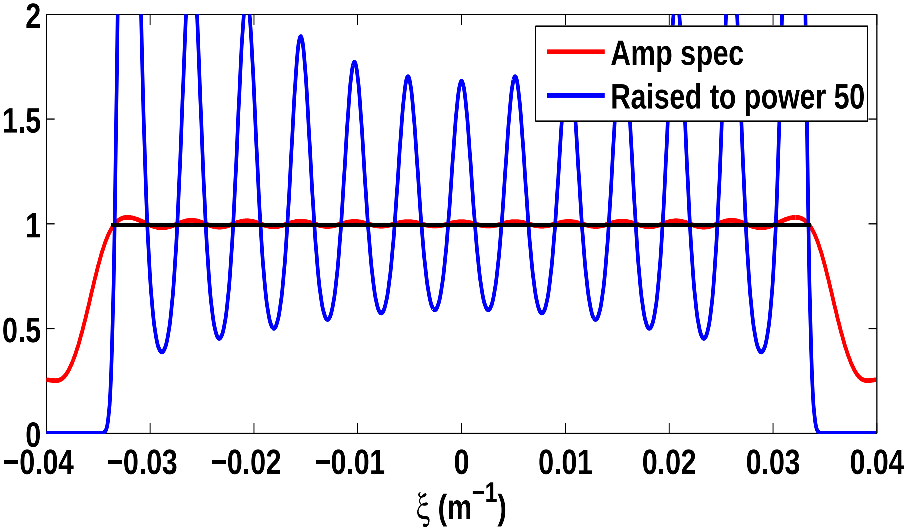

Stabilization by reconstruction

Hale's extrapolator

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- Though effective, it requires significant pen-and-paper analysis to change operational parameters (*eg* changing the spatial sampling of the operator).
- Additionally, it is relatively inaccurate at high-angle propagation.

Stabilization by Weighted Least Squares

The wlsq extrapolator

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- Thorbecke *et al.* (2004) stabilize with a weighted least squares (WLSQ) optimization.
- This method works well, and is simpler than the Hale method.
- However, it still requires an optimization which is somewhat expensive and complicated.

Stabilization by foci

The foci extrapolator

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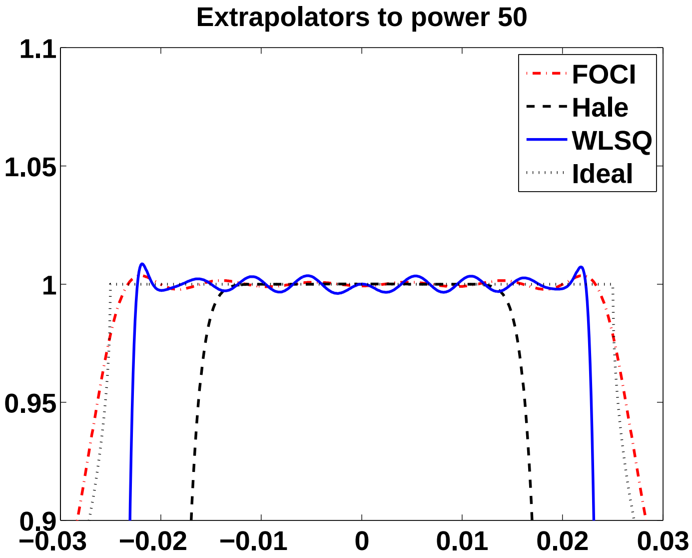
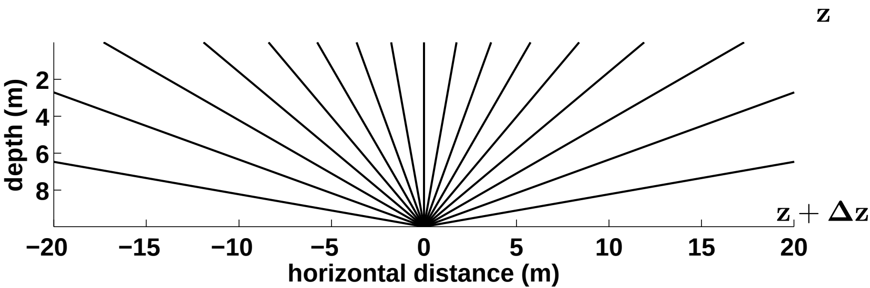
- Compose the forward operator with the conjugate of the inverse to cancel the instability.
- Somewhat complicated to implement correctly.

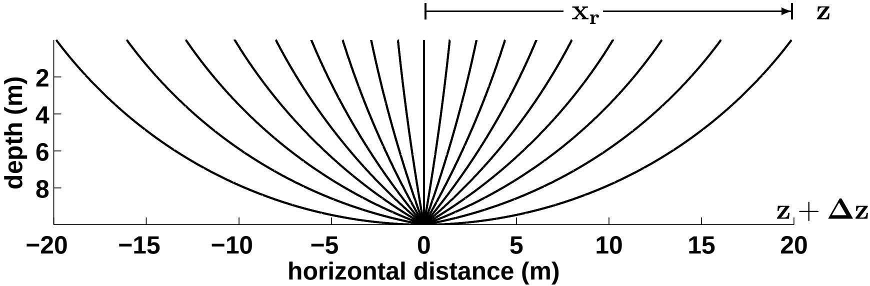
The three extrapolators



The gpspi concept



A different idea



A vertical gradient bends rays!

But what kind of gradient?

- A positive linear vertical velocity gradient, to keep things simple: $v(x,z) = v_0(x) + m(x)(z - z_0), z \in [z_0, z_0 + \Delta z]$.

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- Vertical traveltime should match the original medium.
- Rays leaving at 90° from the output point $(x_0, z_0 + \Delta z)$ should intersect $z = z_0$ at a specified finite aperture radius x_r.

Example parameters

Medium at $v_{loc} = 2000m/s$, for $x_r = 20m$ over $\Delta z = 10m$

- Vertical traveltime match: $\log\left(1 + \frac{m\Delta z}{v_0}\right) = \frac{m\Delta z}{v_{loc}}$.

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- Combining:

$$v_0 = v_{loc} \log\left(1 + \frac{2\Delta z^2}{x_r^2 - \Delta z^2}\right) \frac{x_r^2 - \Delta z^2}{2\Delta z^2}$$

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- In practice: x_r "significantly" bigger than Δz
- Also: x_r "significantly" less than the spatial extent of the convolution kernel
- $x_r/\Delta z \approx [\text{kernel extent}]/x_r$

The lwkbj operator

Developing the lwkbj operator

- Decompose our operator into a composition of N operators:

$$\mathbf{T}_{\alpha(0:\Delta z)} = \mathbf{T}_{\alpha\left((N-1)\frac{\Delta z}{N}\right):\Delta z)} \circ \cdots \circ \mathbf{T}_{\alpha\left(\frac{\Delta z}{N}:2\frac{\Delta z}{N}\right)} \circ \mathbf{T}_{\alpha\left(0:\frac{\Delta z}{N}\right)}$$

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- Use with our definition of $v(x, z)$ to obtain:

$$\alpha(0:\Delta z) \sim \alpha\left(0:\frac{\Delta z}{N}\right)\alpha\left(\frac{\Delta z}{N}:2\frac{\Delta z}{N}\right)\cdots\alpha\left((N-1)\frac{\Delta z}{N}:\Delta z\right)$$

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- Recalling our previous α we can write this as:

$$\alpha(x,\xi,\omega,0:\Delta z) \sim \exp\left(i\frac{\Delta z}{N}\sum_{j=1}^{N}\sqrt{\frac{\omega^2}{v(x,j\Delta z/N)^2}-\xi^2}\right)$$

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- Choosing physically-appropriate branches of square root, and rewriting as a WKBJ-style integrated phase,

$$\alpha \sim \begin{cases} \exp\left(i\int_0^{\Delta z}\sqrt{\frac{\omega^2}{v(x,z')^2}-\xi^2}dz'\right), & |\xi| \leq \frac{|\omega|}{v(x,\Delta z)} \\ \exp\left(-|\int_0^{\Delta z}\sqrt{\frac{\omega^2}{v(x,z')^2}-\xi^2}dz'|\right), & |\xi| > \frac{|\omega|}{v(x_0,\Delta z)} \end{cases}.$$

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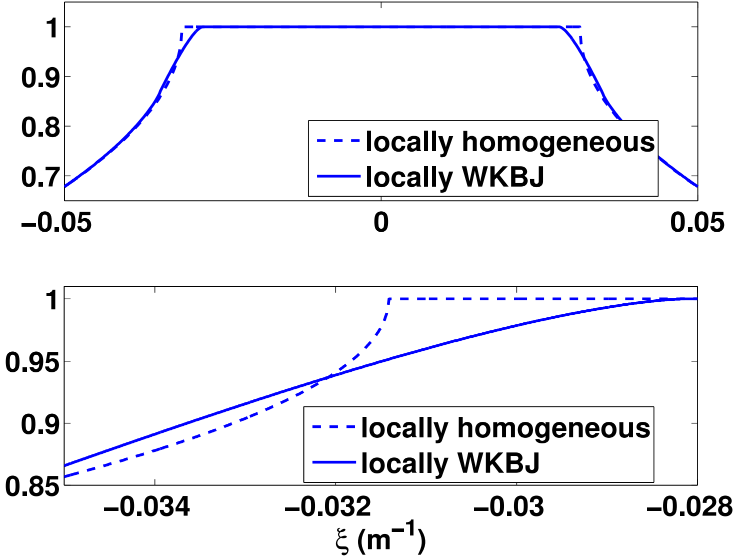
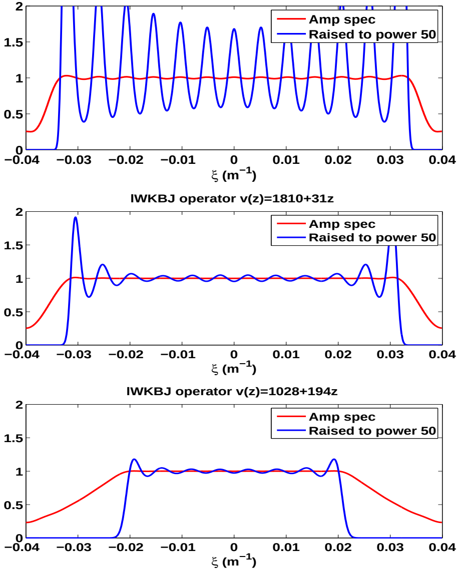
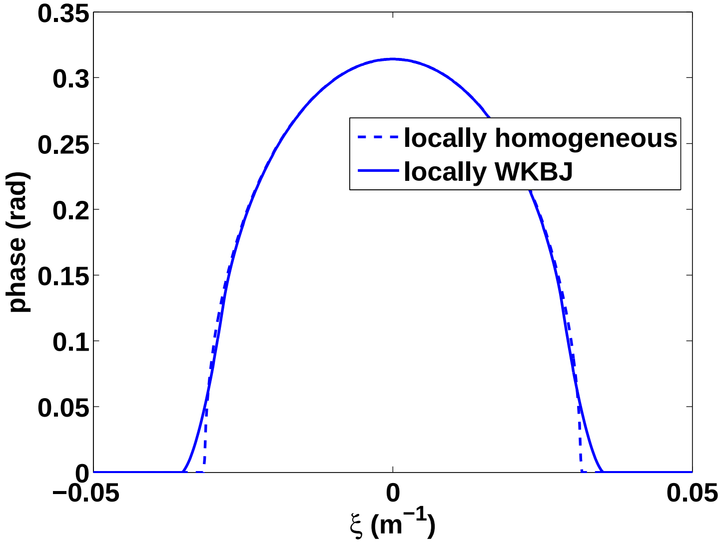
The ideal lwkbj amplitude



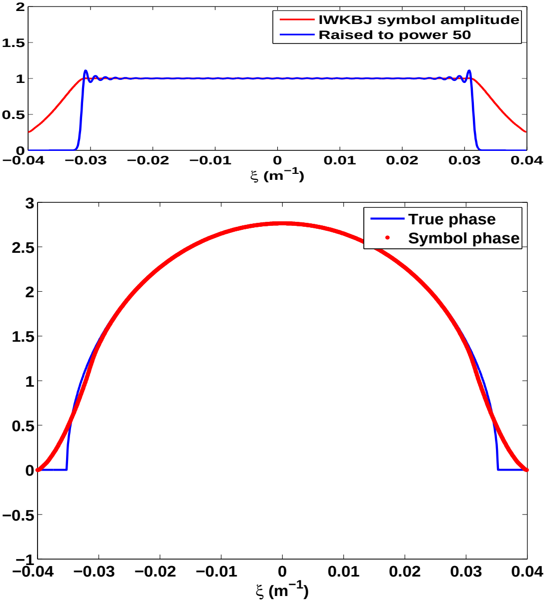
Truncated lWKBJ amplitudes



The ideal lWKBJ phase

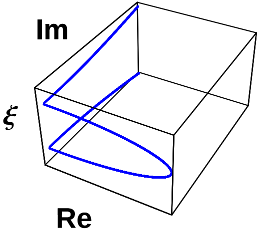


Self-censoring property

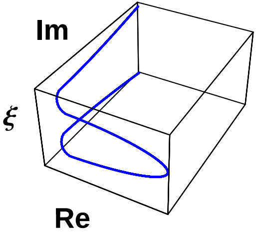


The parametric view

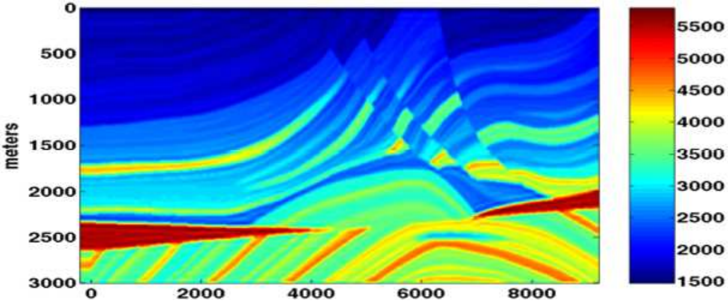
$$\sqrt{\frac{\omega^2}{v(x)^2}-\xi^2}$$



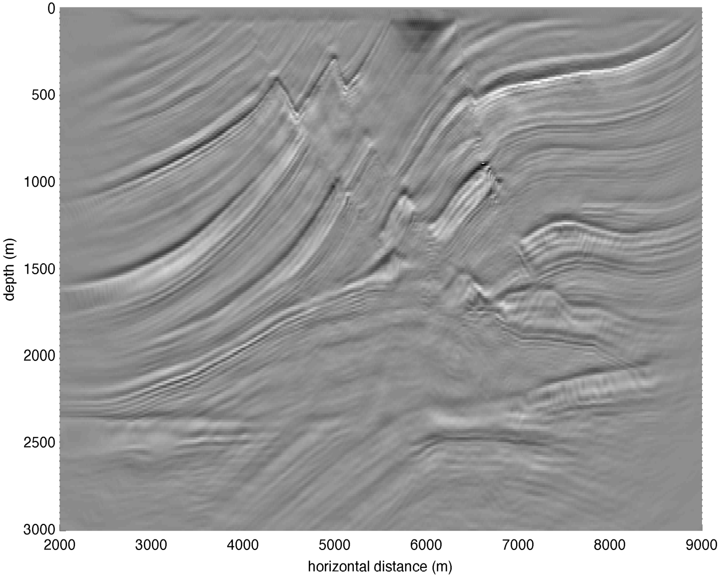
$$\int\sqrt{\frac{\omega^2}{v(x,z)^2}-\xi^2}\,dz$$



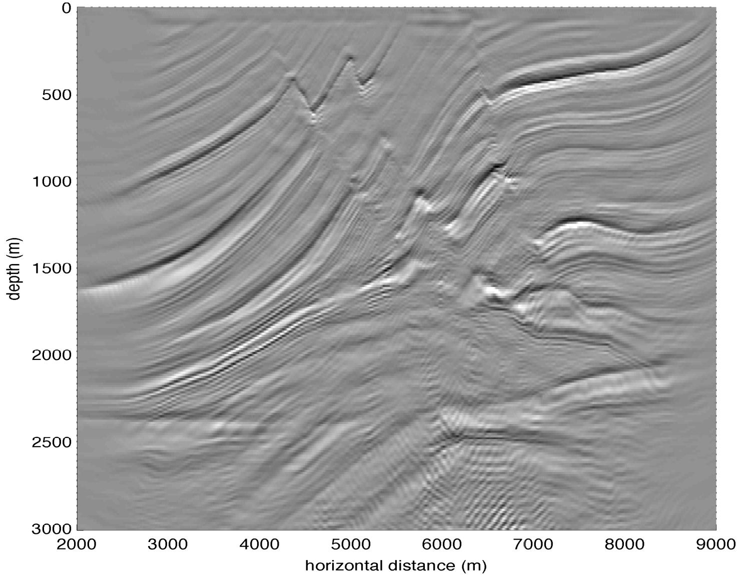
Marmousi imaging



High-resolution image (101 pt operator)



Fast image (15 pt operator)



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- This LWKBJ operator enhances operator stability when spatially localized.
- This operator can be used to migrate very complex images with high accuracy.
- It is extremely simple to implement.

Acknowledgements

- Gary Margrave
- Saleh Al-Saleh
- Hugh Geiger
- Michael Lamoureux

Acknowledgements

POTSI
PSEUDODIFFERENTIAL OPERATOR THEORY & SEISMIC IMAGING

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