Kirchhoff inversion for Incident Waves Synthesized from Common-Shot Data Gathers

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Synthesis:

- Combines common-shot
  data sets
- Survey-wide data sets
- Solution of a single wave
  equation

Synthesis:

- Combines common-shot
  data sets …
  using Green's theorem

Synthesis examples:

- Delayed-shot line source

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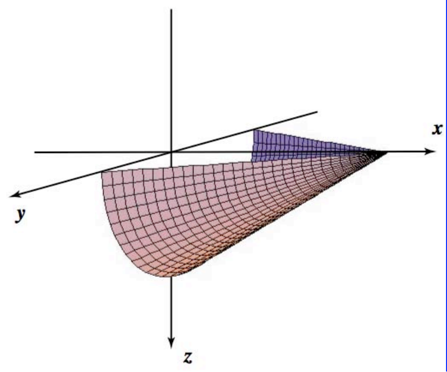
- Delayed shot line source



cone angle

y coordinate

Synthesis examples:

- Delayed shot line source
- Incident plane wave
- Illumination at depth
  (Rietveld, Wapenaar, …)

Common-shot data set

- Observations of a Green's
  function in "free space."

Common-shot data set

$$G_{\mathrm{FS}}(x,x_s,\omega)=G_{\mathrm{D}}(x,x_s,\omega)+G_{\mathrm{U}}(x,x_s,\omega)$$

$G_{\mathrm{D}}(x,x_s,\omega)$: Downgoing point source response

$G_{\mathrm{U}}(x,x_s,\omega)$: Upgoing point source response

Common-shot data set

$$G_{\mathrm{FS}}(\boldsymbol{x},\boldsymbol{x}_s,\omega)=G_{\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}_s,\omega)+G_{\mathrm{U}}(\boldsymbol{x},\boldsymbol{x}_s,\omega)$$

$G_{\mathrm{U}}(\boldsymbol{x}_r,\boldsymbol{x}_s,\omega)$: **Recorded data**

Green's theorem for synthesis of P

$P(x,\ldots,\omega)$ = **volume integral**

+ surface integral

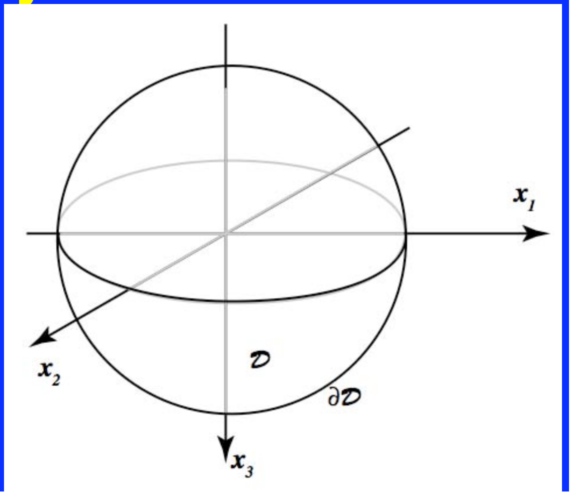
Green's theorem for synthesis

Volume: $\int G \cdot \mathrm{source}$

Surface: $\int G\frac{\partial P}{\partial n}-P\frac{\partial G}{\partial n}$

Green's theorem for synthesis

Volume:



Green’s theorem for synthesis

Volume:

$$P(x,\omega)=\int_{\mathcal{D}} f(x',\omega)G_{\mathrm{FS}}(x,x',\omega)dV'$$

$$f(x',\omega)=\delta(z')F(x',y',0,\omega)$$

$$=\delta(z')F(x',\omega)$$

Green’s theorem for synthesis

Volume:

$$P(x,\omega)=\int_{z'=0} F(x',\omega)G_{\mathrm{FS}}(x,x',\omega)dS'$$

Green’s theorem for synthesis

Volume:

$$P(x,\omega)=\int_{z'=0} F(x',\omega)G_{\mathrm{FS}}(x,x',\omega)dS'$$

$$P(x,\omega)=P_{\mathrm{D}}(x,\omega)+P_{\mathrm{U}}(x,\omega)$$

Green’s theorem for synthesis

Volume:

$$P_{\mathrm{D}}(x,\omega)=\int_{z'=0} F(x',\omega)G_{\mathrm{D}}(x,x',\omega)dS'$$

$$P_{\mathrm{U}}(x,\omega)=\int_{z'=0} F(x',\omega)G_{\mathrm{U}}(x,x',\omega)dS'$$

Green’s theorem for synthesis

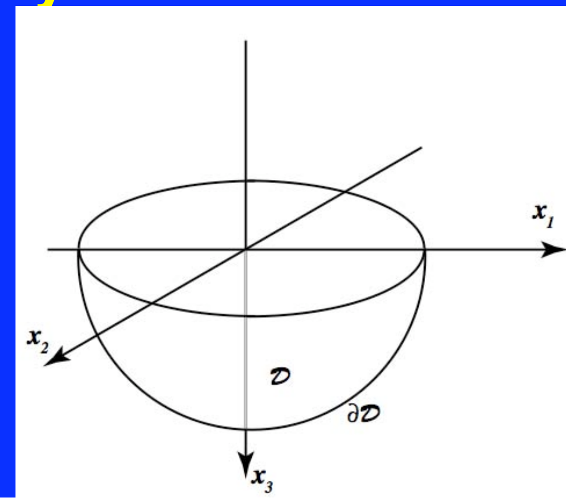
Volume:

$$P_U(x_r,\omega) = \int_{z'=0} F(x',\omega) G_U(x_r,x',\omega)\, dS'$$

Synthesis observed data from common-shot observed data

Green’s theorem for synthesis

Surface:



Green’s theorem for synthesis

Surface: $\int_{z'=0} \left[G \frac{\partial P}{\partial n} - P \frac{\partial G}{\partial n} \right] dS'$

Express G in terms of G_{FS}

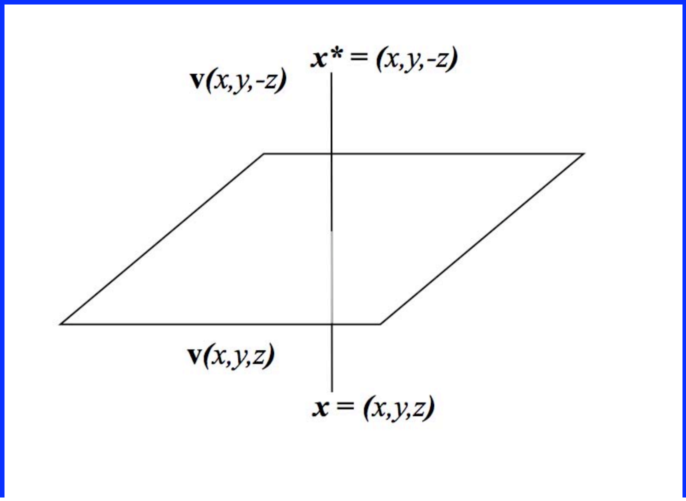
Green’s theorem for synthesis

Surface: $\int_{z'=0} \left[G \frac{\partial P}{\partial n} - P \frac{\partial G}{\partial n} \right] dS'$

Know G, do not know $\frac{\partial G_{FS}}{\partial n}$

Use method of images!

Method of images



Green's theorem for synthesis

Surface:

$$G(x,x',\omega) = G_{FS}(x,x',\omega) + G_{FS}(x^*,x',\omega)$$

Green's theorem for synthesis

Surface:

$$G(x,x',\omega) = G_{FS}(x,x',\omega) + G_{FS}(x^*,x',\omega)$$

x^*: image of x under reflection through $z = 0$

Green's theorem for synthesis

Surface:

$$G(x,x',\omega) = G_{FS}(x,x',\omega) + G_{FS}(x^*,x',\omega)$$

$$\frac{\partial G}{\partial n} = 0, \qquad z = 0.$$

Green's theorem for synthesis

Surface:

$$G(x,x',\omega) = G_{FS}(x,x',\omega) + G_{FS}(x^*,x',\omega)$$

$$G(x,x',\omega) = 2G_{FS}(x,x',\omega), \quad z = 0.$$

Green's theorem for synthesis

Surface:

$$P_D(x,\omega) = -2\int_{z'=0} \frac{\partial P}{\partial n'} G_D(x,x',\omega)dS'$$

$$P_U(x_r,\omega) = -2\int_{z'=0} \frac{\partial P}{\partial n'} G_U(x_r,x',\omega)dS'$$

Green's theorem for synthesis

Surface: synthesized data

$$P_U(x_r,\omega) = -2\int_{z'=0} \frac{\partial P}{\partial n'} G_U(x_r,x',\omega)dS'$$

Modeling:

Downward continue wave fields

- Ray theory
- Gaussian beams
- One-way wave equation
- Two-way wave equation
- Kirchhoff integral

Inversion:

Common-source data set

Inversion output:

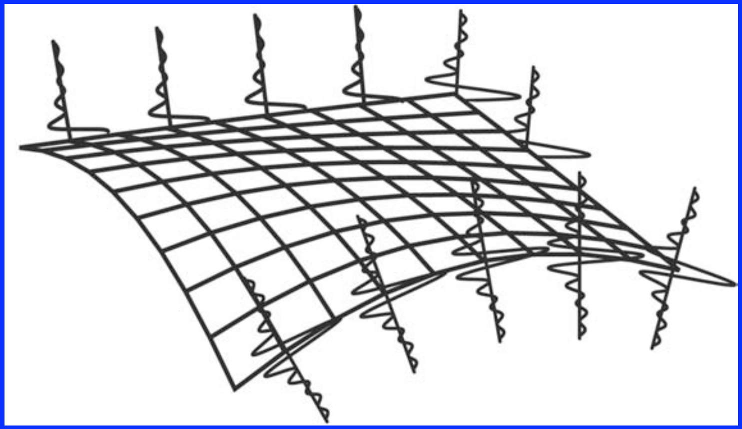
Kirchhoff inversion output

$$\mathcal{R}(x,\theta) = \frac{1}{2\pi}\int \frac{P_U(x,\omega)}{P_D(x,\omega)}\,d\omega$$

Inversion output:

Kirchhoff inversion output

$$\mathcal{R}(x,\theta) \sim R(x,\theta)\gamma(x)$$



$\gamma(x)$

Singular function of a surface

Inversion output:

Kirchhoff inversion output

$$\mathcal{R}(x,\theta)_{peak} = R(x,\theta)\frac{1}{2\pi}\int \mathcal{F}(\omega)d\omega$$

θ : *Specular incidence/ reflection angle*

Illumination at Depth

Travel time is a *line Integral*

Independence of Path!

Illumination at Depth

Independence of Path!

$$\hat{p}_1 \frac{\partial v}{\partial y} = \hat{p}_2 \frac{\partial v}{\partial x}$$

$\hat{p}_1 = \hat{p}_2 = 0$ or $\frac{\partial v}{\partial y} = \frac{\partial v}{\partial x} = 0$

Other topic (in paper)

Commute synthesis, downward continuation

Conclusions

- Response, synthesized source
- Response, synthesized incident wave
- Simple inversion formula

Conclusions

- No Beylkin determinant to compute!