“Finding the tumor”

IAM-CSC-PIMS senior undergraduate math modeling workshop

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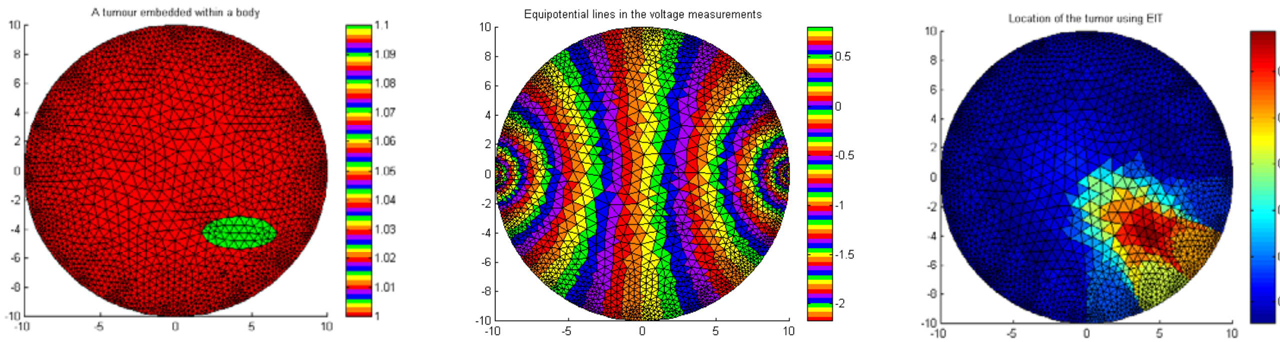

(a) (b) (c)

Finding the tumor: (a) the “tumor” (b) voltage measurements (c) the EIT image

In this project we will introduce some of the mathematics behind Electrical Impedance Tomography (EIT). EIT is a process by which we are able to estimate the electrical conductivity of a body by applying an electrical current through certain points on the boundary of the body and simultaneously performing Voltage measurements at other boundary points. Using these current-Voltage measurements along with a mathematical description of the problem, we are able to arrive at a relatively simple procedure to estimate the conductivity throughout the body being considered.

EIT is a technique which has found application in the field of medical imaging – for example looking for tumors in cancer patients. Tumor cells are known to have very different conductivities from normal cells and can thus be spotted by a procedure that can identify conductivity contrasts within a body. EIT is also applied to geophysical prospecting in which one may wish to identify the region occupied by an ore-body that you wish to extract. By identifying conductivity contrasts between the ore-body and the surrounding rock-mass geophysicists are able to locate desirable mineral deposits without having to drill expensive bore-holes.

Students will be introduced to the basic mathematical equations which govern the physical process involved. Using techniques from undergraduate partial differential equations such as separation of variables, Green’s functions, and Green’s theorem, we will derive the theoretical background for the algorithm. Students will then be shown how to implement the algorithm and we will be able to search for a physical anomaly that has been introduced by simulated measurements. Above we show some results for an example of a circular body: an elliptical “tumor” is introduced into an otherwise homogeneous body (see figure (a)). Making use of the level sets of the appropriate Green’s function (see Figure (b)) we are able to identify (approximately of course) the changes in conductivity between the tumor and the surrounding region (see figure (c)). The procedure does not capture original tumor (shown in figure (a)) precisely - but the resolution in figure (c) is not too poor considering only eight measurements were taken!