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The problem would be to extend the previous PIMS Industrial Problem Solving Workshop IV in 2000 results on Statistical Design of an Experimental Problem in Harmonics in any of several directions that are discussed below.

Original Problem for 2000:
Develop a method that allows estimation of n harmonics on m production steps from sampled waveforms on tires. This method should be flexible, robust and easily constrained to meet operating conditions. (A fuller description is on page 3)

For example we measure a waveform of force variation on a cured tire sampled at 256 equally-spaced points make with 5 products with joint at a fixed angular position. Then we change these relative angular positions, construct a new tire and measure its waveform. We would like to decompose the overall effect represented in each tire into contributions due to each product. We can do this with each harmonic of the waveform but would like to ensure good estimation of all effects for all of a specified set of harmonics.

Proposed problem(s):
The original problem was effectively solved but in so doing there were left unanswered several further issues that would be interesting to pursue:

1) develop fully the method called the Good Lattice Points (GLP) method in the proceedings so that it could be implemented in practice to allow estimation without the prime number restriction

2) include the fitting of a few non-harmonic frequencies with the harmonics and find good designs for these (assumes that all harmonics are not fitted) such as a frequency that passes through the signal exactly 0.62 times

The harmonics are relative to the tire circumference in the old problem, but sometimes effects such as extrusion put sinusoidal patterns into the overall waveform but these effects are not harmonics (the periods are not integral divisors of the tire circumference) of the tire but rather have periods that are fractional parts (like 0.62). We use multiple linear regression to estimate non-harmonic frequency effects and harmonic frequency effects simultaneously (with some correlation between the estimates). The problem is to provide an optimal design strategy for this situation given that we can provide some information like number of non-harmonic frequency effects and possible ranges for their frequencies.

3) best design for a function of the harmonics of different types such as f(type 1 harmonic 1, type 2 harmonic 1) where the experiment is performed on rotation of tire components as before and f is of a specified class or form

In this case we measure two or more type of waveforms on each tire as before. We then combine these multiple outputs into single derived output (often linearly by summation etc. but it could be non-linear). We want to ensure adequate estimation of the product effects for each of a selected set of harmonics for this <u>derived output</u>.

4) expand the concept to a 2 Dimensional Fourier transform or equivalent where the surface could be considered flat or as the surface of an inflated tire (semi-toroidal).

General Statistical Design of Experimental Problem for Harmonics

Submitted by: Michelin Tire

Tires are subjected to a variety of force measurements that are stored as periodic waveforms. Harmonic components of these waveforms are related to tire performance characteristics such as noise and comfort and hence the control and reduction of the amplitudes of these harmonics is an important activity of manufacturing. Technicians may choose to perform designed experiments on their production processes to understand better their impact on the resulting force harmonics. It could be advantageous to have a general design of experiment methodology which allows technicians to choose optimal designs for their studies.

To make this more concrete consider two types of forces (F and G). F is characterized by 5 harmonics F1-F5 and G is characterized by 10 harmonics (G1-G10). Practically the technician might have 20 different process elements (P1-20) that can be rotated within the construction of the tire and which can affect the force measurements. It is assumed that rotation of a production process will result in the equivalent rotation of the force measurement and that superposition of P1 and P2 will result in a corresponding superposition of resulting F1s. In general the movement of any process element such as P1 may affect all harmonics and forces (F1-F5 and G1-G10).

The general problem is to choose the angles of rotation for a set of Ps so that the harmonic effects are well estimated and the cost of experimentation is minimized. Note that the variance of the estimates is related to the angles chosen (for example choosing 180 degrees prevents the estimation of the even harmonics) and that the cost of a study is proportional to the number of angles that are used in the design. Other features of interest include the reparability/extensibility of designs, identifying sets of competitors/surrogates and allowing different precision for different harmonics.