Modeling nonlinear pulse propagation in optical transmission lines

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Abstract

The possibility of sending tremendous amounts of data in the form of light pulses through optical fibers has transformed the world in the recent years. High-speed internet and video on demand are only two examples of exciting new communication technologies. In this workshop, you will learn how to describe optical fiber lines mathematically and discover what are the challenging problems that researchers find themselves engaged nowadays. Throughout this course, you will become familiar with several mathematical tools that are also helpful in many other fields of applied mathematics.

Basic model

The first part of the workshop will introduce you to the physics of light propagating in nonlinear media. From there, we will formulate the basic equation for pulses in fiber lines which is given by the cubic nonlinear Schrödinger equation [4]:

$$iA_z + d(z)A_{tt} + c(z)|A|^2A = 0. \tag{1}$$

This is a nonlinear partial differential equation. Don't worry if you have never seen it before - the aim of the workshop is to learn how to deal with equations like this.

After formulating the mathematical model of the fiber line, we will first focus on relating the parameters that are present in the model (1) to the real physical world [2]. This point constitutes a very important step in the modeling phase. We will follow this with a discussion of several techniques for solving (1). Since we are unable to find a general solution, we will develop strategies to simplify (1) in practically relevant cases. Dispersion-managed fiber networks are the most promising candidates for transmission lines of the next generation. Therefore, we will develop an appropriate reduction of (1) in this case [3]. Here, you will learn how to use multi-scale expansions [1].

After this first part, you will be ready to solve some of the following projects. I know some of the answers, but not all, so we will have to work together.

Projects

- Show that the approximation admits special (beautiful!) solutions (so-called "dispersion-managed solitons") which could be used as bit-carriers. Description of the properties of such solutions.
- Analyze the validity of the chosen approximation for different parameter ranges of real fiber lines.
- Analyze the effect of stochastic perturbations on the original system.
- Find a characterization of the stability of the optical pulses.
- Discussion of pulse interactions.

References

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[3] I. Gabitov and S. K. Turitsyn. Averaged pulse dynamics in a cascaded transmission system with passive dispersion compensation. *Opt. Lett.*, 21:327–329, 1996.

[4] A. Hasegawa and Yuki Kodama. *Solitions in optical communications.* Clarendon Press, Oxford, 1995.