Title : Solving Fredholm integral of the first kind in two dimensions

Fredholm integrals of the first kind are frequently encountered in diverse applications such as image restoration and nuclear magnetic resonance (NMR). The Fredholm integral of the first kind is of the form

In most applications, the kernel is a smooth, continuous function and denotes the joint probability density function of random variables and . For example, in image restoration, the kernels in (1) represent the blurring phenomena. In NMR, the study of two-dimensional functions can provide insight into the molecular processes of relaxation providing an additional dimension to resolve the spin system. Fredholm integrals of this kind are also encountered in solving inverse Laplace transforms.

It is well known that solving Fredholm integrals with smooth kernels is an ill-conditioned problem since the singular values quickly decay to zero. In this problem, we will restrict ourselves to cases where the physics of the problem dictate that the kernel is the tensor product of two kernels and .

In the literature, there have been two methods to solve Fredholm integrals. The first approach uses truncated singular value decomposition. In this formulation, a closed form expression for can be found in terms of the singular values of and . The number of singular values is used to control the quality of the solution. This approach has two main drawbacks that make it unattractive in our application. First, the estimated density function is not necessarily non-negative. Second, choosing the number of singular values usually relies on the availability of stochastic properties of the true density function; these properties are unknown in many applications.
In the second approach, the non-negativity constraint is imposed in a regularization framework. The regularization functional is used to incorporate a priori information about the unknown density function. This a priori information is typically a measure of the smoothness of the desired solution and is weighted by a smoothing parameter. The data can be compressed using singular value decomposition; the estimation problem can then be posed in an optimization framework. The Butler-Reeds-Dawson (BRD) method can then be used to transform the constrained optimization problem into an unconstrained optimization problem in a lower dimensional space.

The workshop can be organized as follows:

Day 1: Explain the basics of the problem; how the problem arises in NMR
Day 2: Provide two solutions to the problem; explain the BRD method to solve problem efficiently in a zero-th order regularizational framework
Day 3 ° 5: Explore other regularizational functionals in BRD framework; this still remains an open problem.
Day 6: Invite workshop participants to share their solutions.