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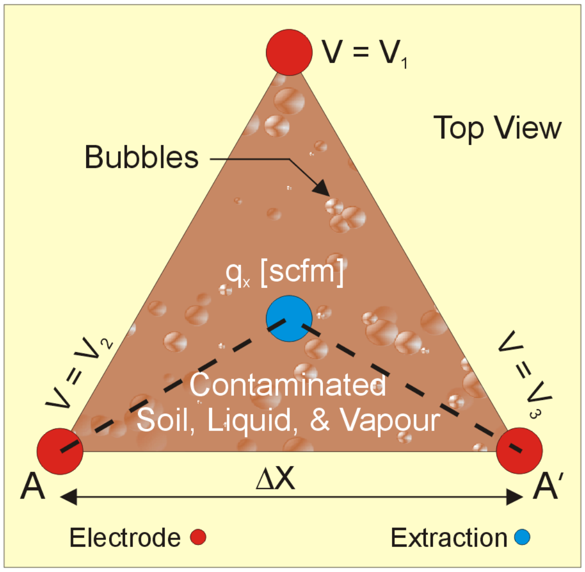
Subject: **Mc^2 Problem for PIMS Workshop in Calgary**

Dr. Rachel Kuske:

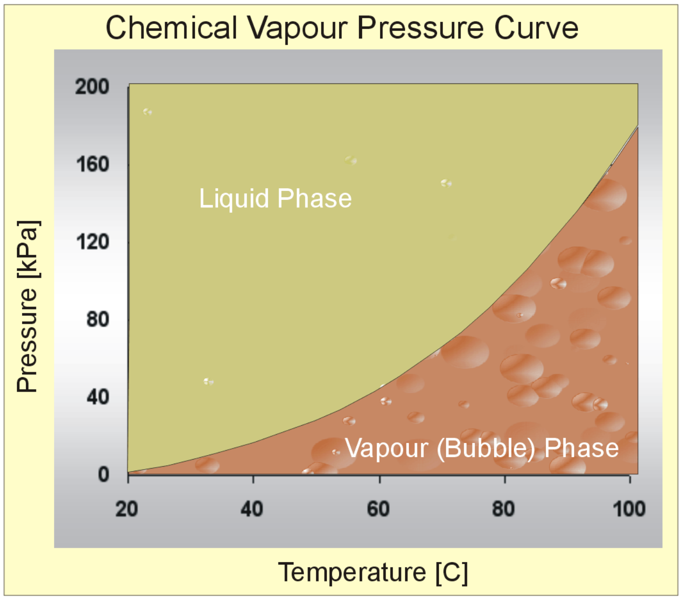
The purpose of this letter is to submit a problem for the PIMS Workshop in Calgary (June, 2003). I shall call this problem; *The Thermodynamic Bubble Problem for the In-Situ Thermal Remediation of Contaminated Soils*.

Electrical energy is introduced to the contaminated soil using a multitude of finite length cylindrical electrodes. Current is forced to flow though the soil by the voltage differentials at the electrodes. The soil is resistive in nature and consequently heat losses occur in accordance with the power density distribution established by the voltage gradient between the electrodes, for example, $q_e = \sigma \mathbf{E} \cdot \mathbf{E}^{\mathbf{T}}$ where $\mathbf{E} = \vec{\nabla} \cdot V$. As the soil heats up contaminated liquids and vapours are produced at the extraction wells. The rate of energy produced from the soil at the extraction wells is less than the electrical energy injected so that during extraction the soil will increase in temperature up to some maximum value, which is usually the boiling temperature of water. The layout of electrodes and extraction wells is shown in the following figure.



The distance between the electrodes is usually seven to eight meters. The distance between the extraction well and an electrode is about four meters. The diameter of the electrodes is 0.2 [m] and 0.1 [m] for the extraction well.

At initial conditions the pressure and temperature in the soil are P_o and T_o respectively. Initially there is no vapour in the soil, only a liquid phase consisting of water and a dissolved chemical phase (the contaminant). As the temperature increases (most rapidly near the electrodes) to $T = T_c$, the critical temperature of the chemicals in the liquid, the chemical *boils* out of the water and exists in the soil as a vapour, but only to the limited extent of several bubbles of some calculated volume. The vapour-equilibrium curve for the chemical is shown in the next figure. Only liquid exits at a temperature and pressure condition above the equilibrium boundary and vapour (bubble) phase below it.



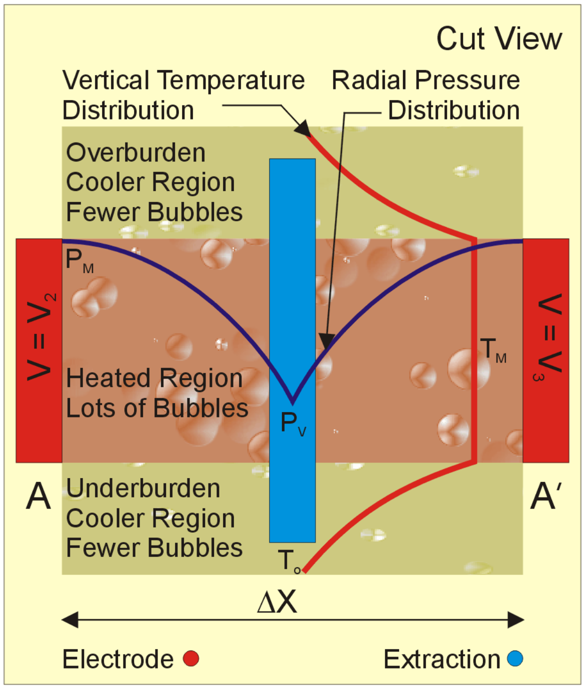
Only a maximum N_B bubbles can exist at any moment. This is due to a volume limitation within the porosity of the soil called the critical gas saturation S_{gc}. No bubbles can exist above S_{gc}. The bubbles can be of different size, which for the purposes of this problem, is determined by the *Ideal Gas Law*, $P_i \cdot V_i = nRT$. The size and number of the bubbles must not exceed the critical gas saturation constraint.

Please refer to the following figure. The vertical temperature distribution in the soil between the electrodes is T_M. The initial temperature in the soil and the temperature far from the heat source is T_o. The temperature declines into the over and under-burden according to:

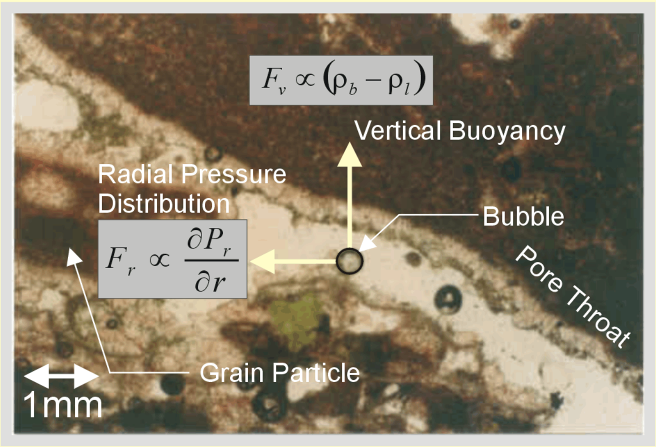
$$T\left(\pm z\right) = T_o + \left(T_M - T_o\right) \cdot \mathrm{erfc}\left(\frac{\pm z}{2\sqrt{\alpha t}}\right) \tag{1}$$

The pressure at the electrodes is P_M and declines to P_V, the vacuum pressure at the extraction wells. The pressure distribution between the electrodes and the extraction well is approximated by the following equation:

$$P\left(r\right) = P_o + \left(\frac{P_V - P_o}{\ln\left({}^{r_e}\!/\!_{r_w}\right)}\right) \cdot \ln\left({}^{r}\!/\!_{r_w}\right) \tag{2}$$



Given the temperature and pressure conditions in the soil it is possible to approximate where the bubbles will exist. The following figure shows a chemical bubble created in the heated porous media (soil) due to its volatilization at temperature and pressure conditions below the equilibrium curve. There is a pressure gradient in the soil created by the extraction well of finite length. This imposes a horizontal force on the bubble. The other force acting on the bubble is buoyancy, which is a function of temperature, (acting in the vertical direction). An equation will need to be derived to determine the momentum or flow path of the bubble in the porous media as it moves about the pressure and temperature distribution through the pore throats (a *chaotic path* with limited statistical description).



Thin Section of Soil Sample

As previously discussed, there is a decreasing temperature gradient along the vertical path of the bubble outside of the heated volume. Therefore as the bubble rises out of the heated volume it flows into cooler and cooler regions. There is a point where the temperature and pressure conditions are such that a thermodynamic event occurs, the bubble becomes a liquid.

The objective is to determine the necessary vacuum pressure so that the chemical bubble is removed at the extraction well (thus cleaning the soil) before it rises too high and condenses back to a liquid. If the bubble does condense, what happens to it? Can the liquid chemical return to the heated zone to become a bubble once again? What are the path options for the thermodynamic bubble with these forces acting on it? Some assumptions are stated below to simplify the problem.

1. Gas phase exits as bubbles. There is no free gas phase.
2. The pressure gradient between the electrode wells and the extraction well is steady state and based on a non compressible fluid system.
3. The soil and liquid properties are homogeneous.
4. The size of the bubbles are a function of pressure and temperature which may be estimated using the *Ideal Gas Law*.
5. Steady state pressure and temperature conditions exist in the soil, initially (the problem may require a non-steady state approach).
6. The temperature at the outer boundaries of the over and under-burden is T_o for the problem.

If you have any questions, please feel free to call me at 403.204.5249 or email mcgee@mcmillan-mcgee.com.

Sincerely

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