Thin Fluid Film Drainage

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1 Introduction: Draining fluid films

Over the last few years, there has been a collaboration between faculty and students at the University of Delaware and chemists and physicists at Dow Corning. The collaboration revolved around the Dow Corning scientists efforts to model fluid film drainage in foams. Those scientists had developed a relatively simple fluid dynamics experiment that was essentially a vertical soap film in a rougly 1 centimeter square window and the bottom of the window was in a bath. Some geometric measurements of the foam drainage were taken, including interferometry (where counting the resulting fringe patterns enabled one to calculate at least relative changes in the film thickness with time). A more sophisticated theory for the film drainage was sought, and most of the mathematical results of the ensuing collaboration is summarized in [1].

As is often the case, more questions arise during the course of research work; some of those questions are proposed as problems here, and they are in the general area of thin fluid film drainage. There are two classifications to draining film problems; in the case of foam, liquid layers have air on both sides, and thus there is a free boundary on both sides of the liquid. (A problem that requires finding the domain as part of the problem is called a "free boundary problem;" see e.g., [2].) This type of fluid film is called a free film. If the thin film is on a wall, it is called a bounded film. For a general overview of thin film research, a recent review can be found in [4]; introductory material on fluid dynamics can be found in [3], e.g. A sketch of a geometry of a bounded film of interest in shown in Figure 1. This geometry was studied in [5] in the presence

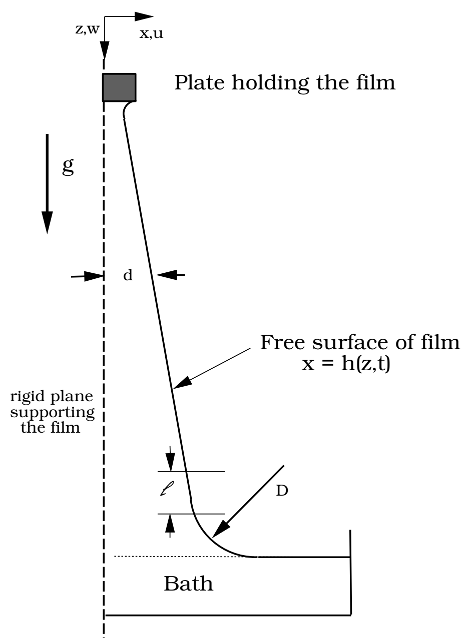

Figure 1: Sketch of a vertical film composed of a single fluid on a substrate; this is the case of a "bounded film". The basic approach for modeling is to take advantage of the thin geometry of the film and use lubrication theory to develop nonlinear PDEs for the free surface given by $x = h(z, t)$.

of a surfactant with various properties. A surfactant is a surface active chemical that changes the surface tension of a fluid surface; the chemical is surface active because its structure causes it to accumulate at the surface of a fluid. This happens because it is energetically favorable for part of the surfactant molecule to be outside of the liquid. I would like to investigate problems that have simpler interfacial properties that those studied in [1, 6].

The fluid layers I would like to study have a small thickness and long extent; this leads to a separation of scales. Using perturbation theory to take advantage of this leads to more tractable problems that describe the evolution of the thin film. In one of the simplest cases, appropriate assumptions and a lubrication (multiple scale) analysis lead to a single equation that describes the evolution of the film's surface $x = h(z, t)$ [1]:

$$h_t = \frac{1}{3}\left[h^3(h_{xxx} + 1)\right]_x = 0.$$

Here h is the film thickness, x and z are spatial coordinates across and along the film respectively, and t is time. This equation, with appropriate boundary and initial conditions, can be studied with numerical methods and with similarity methods.

We will generaize this equation in several ways as time permits. Generally, when we increase the complexity of the problem, there will be additional PDE's and they will be coupled together. In spite of the increase in complexity, we may still be able to profitably use similar approaches to generate useful answers.

2 Problems to Study

I've chosen problems that are extensions to the above. At this time, I don't know all the answers; we'll have to find them together. I expect to do only a subset of the following.

1. Visualization of fringes in the drainage of a thin fluid film. This can be as simple as making a movie of contour plots, but in the case of a free film with deforming surfaces on both sides, the fringe pattern from using interferometry may be a little more complicated than that. A 2D model (film in 3D) could be developed to visualize as well. A single nonlinear pde can be derived and solved in the simplest case.

2. Drainage of a bounded film where the average surface tension is negligible with a soluble surfactant. In this case, computed results are possible, but there may be some similarity analysis that can be carried out [6].

3. Drainage of a two-fluid film, where a second fluid thin film covers the thin film on the rigid wall. Similarity analysis may be possible and numerical computation can certainly be done.

4. Similarity analysis of the region near the top of the film in the single equation case can be carried out. Numerical solution indicates a power law behavior for the minimum film thickness and preliminary analysis agrees with the numerics, but there are a number of issues to resolve in the similarity analysis.

References

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