Seismic inverse problem in anisotropic, inhomogeneous media

The solutions of many seismic problems consist of obtaining the information about the subsurface based on the surface measurements of seismic signals. Also, we attempt to obtain this information based on the measurements of a seismic signal between the source on the surface and the receiver in the wellbore. Such a method is referred to as the vertical seismic profile (VSP). The key measurements consist of traveltime and amplitude of the signal. In the recent past, however, the technological advances also allow us to measure the angle of arrival of the signal.

Many seismic problems are treated in the context of the asymptotic ray theory. This is a high-frequency approximation to the wave theory. In the context of the ray theory, we assume that the signal travels along the ray, which is the trajectory of the stationary traveltime; in other words, seismic signal obeys variational principle of Fermat.

Most seismic problems are formulated in the context of continuum mechanics. In other words, the medium through which seismic waves propagate is assumed to be a continuum. Furthermore, the common constitutive equation is a linear stress-strain relation, namely, Hooke's law. Consequently, a seismic medium is completely described by its mass density and elasticity constants. In seismic theory, velocity of the signal along the ray is a function of mass density and elasticity constants.

In this problem, following the variational principle of Fermat, we wish to obtain as much information as possible (mass densities, elasticity constants) about a layered medium through which seismic signals propagate. Our information consists of the traveltime and arrival-angle measurements obtained through VSP. Our medium is anisotropic and, consequently, we must carefully treat such quantities as phase velocity and ray velocity, as well as phase angle, ray angle and polarization angle. For an isotropic case many of these quantities coincide. In our measurements, however, the signal travels along to the ray angle while the arrival angle is measured in terms of the polarization angle since, in the anisotropic media, these quantities are distinct.