

# Locating Watchtowers in Terrains

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We focus on the watchtower problem which in general can be stated as follows. A polyhedral terrain (mountain landscape) in three dimensions is a continuous bivariate function  $z = T(x, y)$  which happens to be piecewise linear. A two-dimensional terrain is a  $x$ -monotone polygonal chain. We want to locate  $k$  vertical segments (watchtowers) erected on a terrain such that every point on the terrain is visible from the top of either watchtowers and the maximum height of watchtowers is minimized. The problem arises in Geographic Information Systems and Facility Location.

The watchtower problem can be viewed as generalization of the problem of guarding terrains where one want to decide whether a set of  $k$  points on a terrain (which can be thought as watchtowers of zero height) can collectively guard the terrain. The problem of guarding terrains is NP-hard. However, for fixed number of watchtowers, almost nothing is known. Sharir'88 obtained  $O(n \log^2 n)$  algorithm for one watchtower in 3D. Zhu'97 improved the running time to  $O(n \log n)$ . There are some results for locating two watchtowers in the plane.

The main question is to characterize the solution for two watchtowers in 3D. Another question is to design an algorithm for 3 watchtowers in the plane.