Spectral Methods for Discontinuous Probl

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Plan of the Talk

- Spectral Galerkin and Collocation Schemes.
- Approximation Theory: The Gibbs phenomenon
  - The resolution of the Gibbs Phenomenon.
  - Application: Splicing of pictures.
  - Application: The Direct Fourier Method in Computed Tomography.
- Linear Hyperbolic equations with discontinuous solutions.
  - Stability and filtering.
  - Recovering spectral accuracy.
- Non linear Hyperbolic Equations:
  - The Spectral Viscosity Method
  - The Super Viscosity Methods and its relationship to filtering.

Plan of the Talk (Cont.)

- Applications:
  - The Leblanc Problem.
  - Interactions of shock waves and hydrogen jets in reactive flows.
  - Recessed cavity flameholders.
  - Richtmyer-Meshkov instabilities.

Spectral Methods - Introduction

The solution of an equation is assumed to be in a space $\mathcal{B}_{\mathcal{N}}$ spanned by smooth

- For periodic problems

$$\phi_k = e^{ikx}$$

- for non-periodic problems in finite intervals
  - Either

$$\phi_k = T_k(x)$$

    where T_k are the Chebyshev polynomials.
  - Or

$$\phi_k = L_k(x)$$

    where L_k are the Legendre polynomials.

The approximation is obtained in one of two ways:

- Galerkin projection:

$$\mathcal{P}_{\mathcal{N}} F(x) = \sum_{k=0}^{N} (F, \phi_k) \phi_k(x)$$

  where

$$(F, \phi_k) = \int \omega(x) F(x) \phi_k(x) \; dx$$

- The Pseudo-spectral approximation

$$\mathcal{I}_{\mathcal{N}} F(x) = \sum_{k=0}^{N} (F, \phi_k)_N \phi_k(x)$$

where

$$(F, \phi_k)_N = \sum_{i=0}^{N} \omega_i F(x_i) \phi_k(x_i)$$

  - x_i and ω_i are the Gauss-Lobatto Quadrature nodes and weights respec
  - Thus

$$\int f(x)\omega(x)dx = \sum_{i=0}^{N} \omega_i f(x_i)\phi_k(x_i)$$

    when $f(x)$ is a polynomial or trigonometrical polynomial of degree $2N$

- Alternatively

$$\mathcal{I}_{\mathcal{N}} F(x) = \sum_{i=0}^{N} F(x_i) g_i(x)$$

where

  - $g_i(x)$ is a polynomial of degree N such that $g_i(x_k) = \delta_{i,k}$.

This leads naturally to the formula:

$$\frac{d}{dx} \mathcal{I}_{\mathcal{N}} F(x_k) = \sum_{i=0}^{N} F(x_i) g_i'(x_k)$$

and defines the *Differentiation Matrix*

$$\mathcal{D} = g_i'(x_k)$$

Approximation Results

$$||F - \mathcal{P}_N F|| \leq K \frac{||F||_s}{N^s}$$

$$||F - \mathcal{I}_N F|| \leq K \frac{||F||_s}{N^s}$$

Approximation Theory

- We assume that
  - $f(x)$ is in $L^2[-1,1]$;
  - there is a subinterval $[a,b] \subset [-1,1]$ in which $f(x)$ is analytic;
  - there exists an orthonormal family $\{\Psi_k(x)\}$, under a scalar product $(\cdot$
- Denote

$$f_N(x) \; = \; \sum_{k=0}^{N} (f, \Psi_k)\Psi_k(x)$$

-

$$\lim_{N\to\infty} |f(x) - f_N(x)| \; = \; 0$$

almost everywhere in $x \in [-1,1]$.

- Denote $\xi = -1 + 2\frac{x-a}{b-a}$ such that if $a \le x \le b$ then $-1 \le \xi \le 1$.

Definition:

The two parameters family $\{\Phi_k^\lambda(\xi)\}$ is called a **Gibbs complementary** to t

(a) **Orthogonality**

$$< \Phi_k^\lambda(\xi), \Phi_l^\lambda(\xi) >_\lambda = \delta_{kl}.$$

(b) **Spectral Convergence**

The expansion of an analytic function $g(\xi)$ in the basis $\Phi_k^\lambda(\xi)$ converges ex

$$\max_{-1\le\xi\le1} \left| g(\xi) - \sum_{k=0}^{\lambda} < g, \Phi_k^\lambda) >_\lambda \Phi_k^\lambda(\xi) \right| \le e^{-q_1\lambda}, \qquad q_1 > 0$$

(c) **The Gibbs Condition**

There exists a number $\beta < 1$ such that if $\lambda = \beta N$ then

$$\left| < \Phi_l^\lambda(\xi), \Psi_k(x(\xi)) >_\lambda \right| \max_{-1\le\xi\le1} \left| \Phi_l^\lambda(\xi) \right| \le \left(\frac{\alpha N}{k} \right)^\lambda, \qquad k > N,\ l \le$$

Comments:

- Condition (b) implies that the expansion of a function g in the basis $\{\Phi_l^{\lambda}$
nentially fast if g is analytic in $-1 \leq \xi \leq 1$ (corresponding to $a \leq x \leq b$).
- Condition (c) states that the projection of $\{\Psi_k\}$ for large k on the low mo
small l) is exponentially small *in the interval* $-1 \leq \xi \leq 1$.

Theorem

- $f(x) \in L^2[-1,1]$ and analytic in $[a,b] \subset [-1,1]$.
- $\{\Psi_k(x)\}$ is an orthonormal family with the inner product $(\cdot\,,\,\cdot)$.
- $\{\Phi_k^\lambda(\xi)\}$ is a Gibbs complementary to the family $\{\Psi_k(x)\}$ as defined in (a

Then

$$\max_{a \le x \le b} \left| f(x) - \sum_{l=0}^{\lambda} < f_N, \Phi_l^\lambda >_\lambda \Phi_l^\lambda(\xi(x)) \right| \le e^{-qN}, \qquad q > 0.$$

Comment:

- Even if we have a slowly converging series

$$\sum_{k=0}^{N}(f,\Psi_k)\Psi_k(x)$$

it is still possible to get a rapidly converging approximation to $f(x)$ if one c
function that yields a rapidly converging series to f as long as *the projectio*
in the basis $\{\Psi\}$ *on the low modes in the new basis is exponentially s*

Examples

- In all the following examples, we choose

$$\Phi_k^\lambda(\xi) \;=\; \frac{1}{\sqrt{h_k^\lambda}} C_k^\lambda(\xi)$$

where $C_k^\lambda(\xi)$ is the Gegenbauer polynomial and h_k^λ is the normalization f
inner product is defined by

$$< f, g >_\lambda = \int_{-1}^{1} (1-\xi^2)^{\lambda-\frac{1}{2}} f(\xi) g(\xi)\, d\xi$$

We have to check only if:

- $$\left| \int_{-1}^{1} (1-\xi^2)^{\lambda-\frac{1}{2}} C_l^\lambda(\xi) \Psi_k(x(\xi))\, d\xi \right| \leq \left(\frac{\alpha N}{k}\right)^\lambda,$$

  for $k > N, l \leq \lambda = \beta N, 0 < \alpha < 1.$

Example 1: Fourier Case

$$\Psi_k(x) = \frac{1}{2}e^{ik\pi x}, |k| \leq \infty\,.$$

We would need to verify that

$$\left|\int_{-1}^{1}(1-\xi^2)^{\lambda-\frac{1}{2}}e^{ik\pi x(\xi)}C_l^\lambda(\xi)\,d\xi\right| \leq \left(\frac{\alpha N}{k}\right)^\lambda,$$

for $k > N, l \leq \lambda = \beta N, 0 < \alpha < 1$.
There is an explicit formula for the integral

$$\int_{-1}^{1}(1-\xi^2)^{\lambda-\frac{1}{2}}e^{ik\pi x(\xi)}C_l^\lambda(\xi)\,d\xi = h_l^\lambda\Gamma(\lambda)\left(\frac{2}{\pi k\epsilon}\right)^\lambda i^l(l+\lambda)J_{l+\lambda}(\pi$$

where $\epsilon = b - a$ and $J_\nu(x)$ is the Bessel function.
The Gibbs condition is satisfied when

$$\beta = \frac{2\pi\epsilon}{27}.$$

Example 2: Legendre Case

$$\Psi_k(x) = \frac{1}{\sqrt{h_k^{\frac{1}{2}}}} C_k^{\frac{1}{2}}(x).$$

We need to verify that

$$\frac{1}{\sqrt{h_k^{\frac{1}{2}}}} \left| \int_{-1}^{1} (1-\xi^2)^{\lambda-\frac{1}{2}} C_k^{\frac{1}{2}}(x(\xi)) C_l^{\lambda}(\xi)\, d\xi \right| \le \left(\frac{\alpha N}{k} \right)^{\lambda},$$

for $k > N, l \le \lambda = \beta N, 0 < \alpha < 1$.

This had been verified.

Example 3: Gegenbauer Case

$$\Psi_k(x) = \frac{1}{\sqrt{h_k^\mu}} C_k^\mu(x).$$

We need to verify that

$$\frac{1}{\sqrt{h_k^\mu}} \left| \int_{-1}^{1} (1-\xi^2)^{\lambda-\frac{1}{2}} C_k^\mu(x(\xi)) C_l^\lambda(\xi)\, d\xi \right| \leq \left(\frac{\alpha N}{k} \right)^\lambda,$$

for $k > N, l \leq \lambda = \beta N, 0 < \alpha < 1$.

We have to recover a density function $f(x, y)$ from its Radon Transform $p(r, \theta$

$$p(r, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\delta(x\cos(\theta) + y\sin(\theta) - r)\, dx\, dy$$

The Slice Theorem:

$$\hat{p}(\rho, \theta) = \hat{f}(\rho\cos\theta, \rho\sin\theta)$$

where

- $\hat{p}$ is the Fourier Transform (in r) of p,
- $\hat{f}$ is the two dimensional Fourier Transform of f.

The DFM

$$p \rightarrow \hat{p} \rightarrow \hat{f} \rightarrow f$$

Consider first approximating of

$$\frac{\partial U}{\partial t} = \frac{\partial U}{\partial x}$$

by the Fourier method.

Denote:

- $u_N(x)$ is the PS solution satisfying

$$\frac{\partial u_N}{\partial t}(x_l) = \sum_{i=0}^{N} g_i^{'}(x_l) u_N(x_i)$$

- $\vec{u}_i = u_N(x_i)$.

The PS can be written as

$$\frac{d}{dt}\vec{u} = \mathcal{D}\vec{u} \quad .$$

Since $\mathcal{D}$ is anti-symmetric, stability is assured.

The variable coefficient case

Consider now

$$\frac{\partial U}{\partial t} = a(x)\frac{\partial U}{\partial x}$$

- The matrix $\mathcal{AD}$ is not skew symmetric.

How to stabilize?

- Rewrite the equation as

$$\frac{\partial U}{\partial t} = \frac{1}{2}a(x)\frac{\partial U}{\partial x} + \frac{1}{2}\frac{\partial a(x)U}{\partial x} - \frac{1}{2}a'(x)\frac{\partial U}{\partial x}$$

  Doubles the amount of work. Loses conservation.

Analysis:

Write the equation as follows

$$\begin{aligned}\frac{\partial u_N}{\partial t} &= \mathcal{I}_N a(x) \frac{\partial u_N}{\partial x} \\ &= N_1 + N_2 + N_3\end{aligned}$$

where

$$\begin{aligned}2N_1 &= \frac{\partial \mathcal{I}_N a u_N}{\partial x} + \mathcal{I}_N \left(a(x) \frac{\partial u_N}{\partial x} \right) \\ 2N_2 &= \mathcal{I}_N \left(a(x) \frac{\partial u_N}{\partial x} \right) - \mathcal{I}_N \frac{\partial a u_N}{\partial x} \\ 2N_3 &= \mathcal{I}_N \frac{\partial a u_N}{\partial x} - \frac{\partial \mathcal{I}_N a u_N}{\partial x}\end{aligned}$$

Now

- $$(u_N, N_1 u_N) = 0.$$

  Since $\mathcal{AD} + \mathcal{DA}$ is skew symmetric.

- $$|(u_N, N_2 u_N)| \leq K \max_{0 \leq x \leq 2\pi} |a^{'}|(u_N, u_N)$$

- $$|(u_N, N_3 u_N)| \leq (u_N, (-1)^{p+1} \epsilon_N \frac{\partial^{2p} u_N}{\partial x^{2p}})$$

  where $\epsilon_N \sim N^{1-2p}$

Conclusion:

To stabilize the PS method we can write

$$\frac{\partial u_N}{\partial t} = \mathcal{I}_N a(x) \frac{\partial u_N}{\partial x} + (-1)^{p+1} \epsilon_N \frac{\partial^{2p} u_N}{\partial x^{2p}}$$

Consider

$$\frac{\partial u_N}{\partial t} = (-1)^{p+1} \epsilon_N \frac{\partial^{2p} u_N}{\partial x^{2p}}$$

Let

$$u_N = \sum_{k=-N}^{N} a_k(t) e^{,ikx}$$

Then we have

$$\frac{da_k(t)}{dt} = (-1)^{p+1} \epsilon_N (ik)^{2p} a_k(t)$$

Thus

$$a_k(t + \Delta t) = e^{-\epsilon_N k^{2p} \Delta t} a_k(t) \qquad (\textit{This is a low pass filter!!!})$$

Important:

The dissipation in the Legendre case will be therefore

$$(-1)^{p+1}\left(\frac{\partial}{\partial x}(1-x^2)\frac{\partial}{\partial x}\right)^p$$

Linear Hyperbolic Equations

Consider the hyperbolic system of the form

$$\frac{\partial U}{\partial t} = \mathcal{L}U$$

with initial conditions

$$U(t = 0) = U_0$$

- Example: The symmetric hyperbolic system,

$$\frac{\partial U}{\partial t} = \sum_{i=1}^{d} A_i(x_1, ..., x_d)\frac{\partial U}{\partial x_i}$$

Let u be the Fourier Galerkin approximation:

$$(U - u, e^{ikx}) = 0 \qquad -N \leq k \leq N$$
$$(U_0 - u_0, e^{ikx}) = 0 \qquad -N \leq k \leq N$$

Theorem

$$||U - u|| \leq K||U_0||_s \frac{1}{N^{s-1}}$$

This fails for piecewise smooth U_0. This is not surprising since U is not a smoot

Theorem

Let ϕ be a smooth function, then

$$|(U(T) - u(T), \phi)| \leq K||\phi||_s \frac{1}{N^s}$$

Proof

We consider the adjoint problem:

$$V_t = \mathcal{L}^* V$$

with the smooth initial condition:

$$V(t = 0) = \phi$$

The Green's identity is derived from

$$\frac{d\,(U(t),V(T-t))}{dt} = (\mathcal{L}U(t),V(T-t)) - (U(t),\mathcal{L}^*V(T-$$
$$= 0$$

So that

$$(U(T),\phi) = (U_0,V(T))$$

We can also show that

$$(u(T),\phi_N) = (u_0,v(T))$$

where v is the Galerkin approximation to V.

- Since the problem for V is smooth then

$$||V(T) - v(T)|| \leq K \frac{||\phi||_s}{N^{s-1}}$$

-

$$(u_0, v(T)) = (U_0, v(T))$$

-

$$(u(T), \phi_N) = (u(T), \phi)$$

Thus

$$(U(T) - u(T), \phi) \;=\; (U_0, V(T) - v(T))$$

Comments

- Similar Results are obtained for the collocation (Pseudospectral) method. H initial condition (u_0) has to be the Galerkin approximation to the initial c
- The Fourier coefficients of u approximate those of U with spectral accu possible to postprocess to get spectral accuracy for the point values.
- Alternatively, define

$$\phi = \sum (1-\eta^2)^{\lambda-\frac{1}{2}} C_k^\lambda(\eta) C_k^\lambda(\xi)$$

for any interval of smoothness, and ϕ vanishes outside.

Then

$$(u,\phi) = u(\xi) \qquad \text{with } \textit{spectral} \text{ accuracy.}$$

Nonlinear Equations

Consider

$$\frac{\partial U}{\partial t} + \frac{\partial f(U)}{\partial x} = 0$$

The spectral method is

$$\frac{\partial u_N}{\partial t} + \frac{\partial \mathcal{P}_N f(u_N)}{\partial x} = 0$$

The method is **unstable** (as in the linear case!)

In the **Spectral Viscosity Method (SV)**

$$\frac{\partial u_N}{\partial t} + \frac{\partial \mathcal{P}_N f(u_N)}{\partial x} = \epsilon_N (-1)^{s+1} \frac{\partial^s}{\partial x^s} \left[Q_m(x,t) * \frac{\partial^s u_N}{\partial x^s} \right]$$

$$\frac{\partial u_N}{\partial t} + \frac{\partial \mathcal{P}_N f(u_N)}{\partial x} = \epsilon_N (-1)^{s+1} \frac{\partial^s}{\partial x^s}\left[Q_m(x,t) * \frac{\partial^s u_N}{\partial x^s}\right]$$

The operator Q_m can be expressed in the Fourier space:

$$\epsilon_N (-1)^{s+1} \frac{\partial^s}{\partial x^s}\left[Q_m(x,t) * \frac{\partial^s u_N}{\partial x^s}\right] \sim \epsilon \sum_{m<|k|<N} (ik)^{2s} \hat{Q}_k \hat{u}_k e^{ikx}$$

with

$$\epsilon \sim C N^{2s-1}; \qquad m \sim N^{\theta}, \;\; \theta < \frac{2s-1}{2s}; \qquad 1 - \left(\frac{m}{|k|}\right)^{\frac{2s-1}{\theta}} \leq \hat{Q}_k$$

In the **Super viscosity**

$$\epsilon (-1)^{s+1} \frac{\partial^{2s}}{\partial x^{2s}}$$

Theorem:

Consider the Fourier Superviscosity method, subject to L^∞ initial data. Ther
u_N converges to the entropy solution.

Theorem:

Let u_N be the solution of the SV approximation, subject to bounded initial co

$$||u_N(,0)||_{L^\infty} + \epsilon^s||\partial^s u_N(,0)||_{L^2} \leq Const.$$

then u_N converges strongly to the unique entropy solution.

LeBlanc Problem

The one-dimensional Euler equation

$$\begin{aligned} \rho_t + (\rho u)_x &= 0 \\ (\rho u)_t + (\rho u u)_x &= 0 \\ E_t + ((E+P)u)_x &= 0 \end{aligned}$$

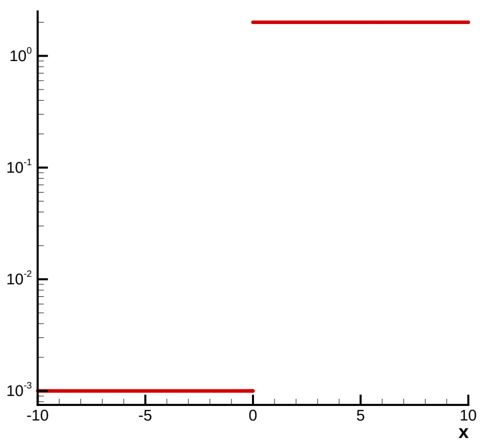
with

$$E = \frac{P}{\gamma - 1} + \frac{1}{2}\rho u^2$$

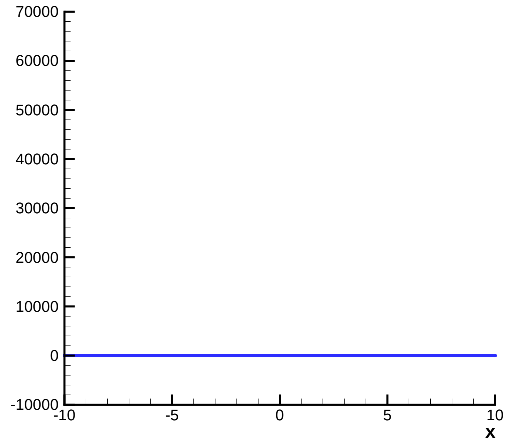
and $\gamma = 1.4$.

The initial condition is the Riemann data :

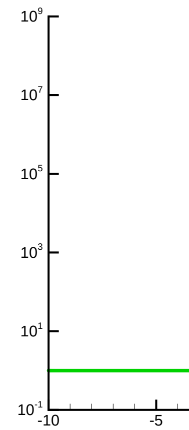
Density



Velocity



Pres



$$\rho = 2, \quad u = 0, \quad P = 10^9 \qquad -10 <= x <= 0$$
$$\rho = 10^{-3}, \quad u = 0, \quad P = 1 \qquad 0 < x <= 10$$

and $t_{final} = 10^{-4}$.

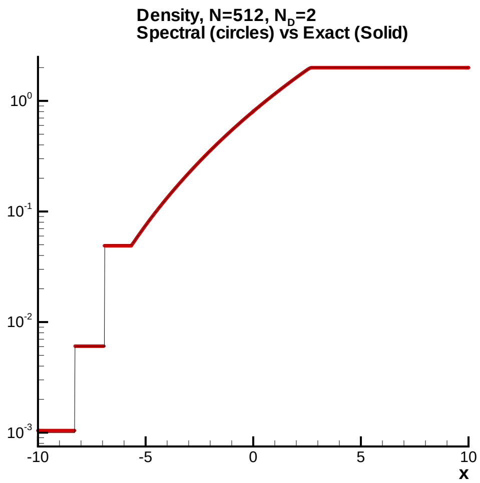
Algorithms :

- Spatial Algorithm :
  1. Multi-Domain Chebyshev collocation method (Spectral),
     - Differentiation and Smoothing operations are done via an optimized
       (Costa & Don);
     - a 9'th order exponential filter used to stabilize the scheme.
     - Riemann Solver applied at the domain interface.
     - The non-oscillatory solutions are obtained by post-processing techni
       Gottlieb).
  2. WENO fifth order finite difference scheme (WENO) with Lax-Frederick
- Temporal Algorithm :
  Third order TVD Runge Kutta method (Shu and Osher).

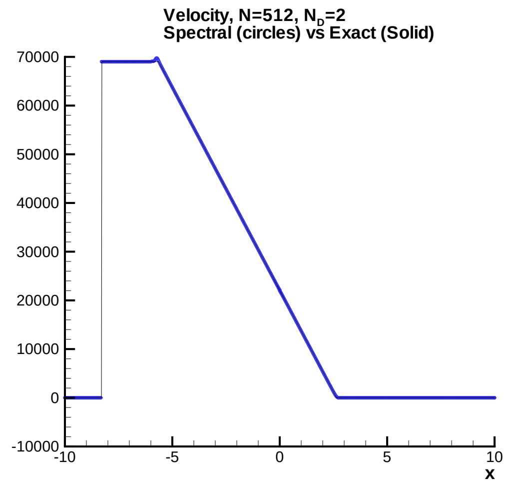
LeBlanc Problem (Cont.)

Spectral Scheme

Density



Velocity



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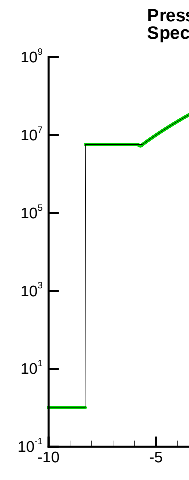
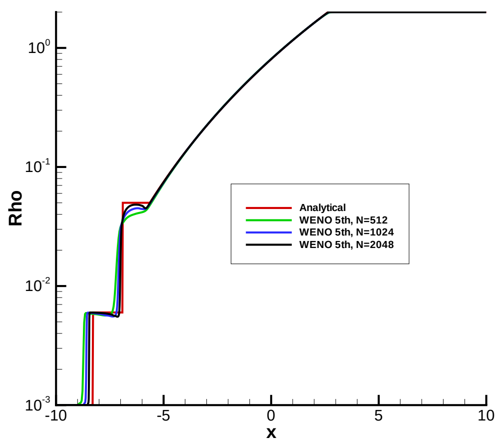


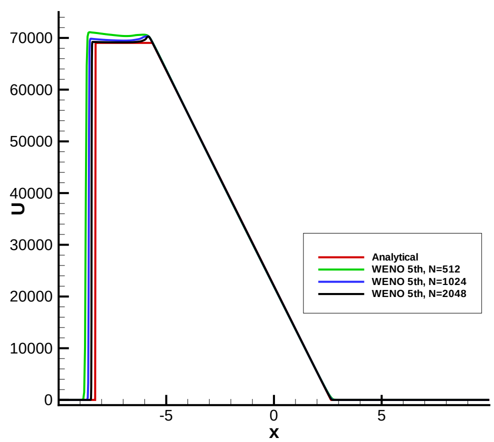
LeBlanc Problem (Cont.)

WENO Scheme

Density



Velocity



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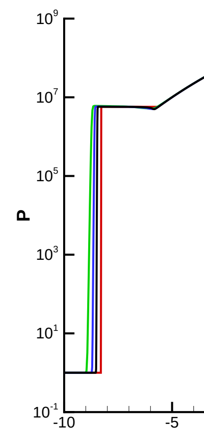


Richtmyer-Meshkov Instability

- Shock induced instability of the interface between fluids of different den Richtmyer and experimented by Meshkov.)
- Growth of the interface amplitude and secondary shear instability promot lence mixing.
- Applications included but not limited to mixing enhancement and inertial

The two-dimensional Euler equations,

$$\mathbf{Q}_t + \mathbf{F}_x + \mathbf{G}_y = 0.$$

The state vector $\mathbf{Q}$ is

$$\mathbf{Q} = (\rho, \rho u, \rho v, E)^{\mathrm{T}} \; .$$

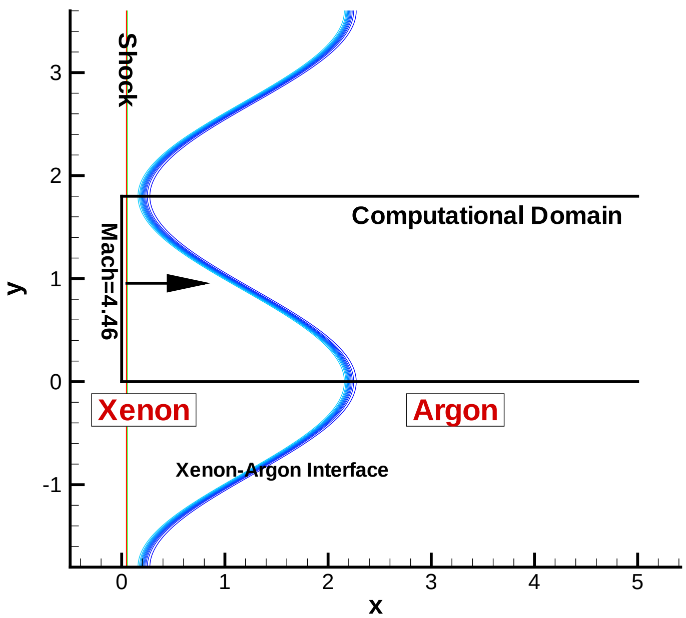
The inviscid fluxes $\mathbf{F}$ and $\mathbf{G}$ are given by

$$\begin{aligned} \mathbf{F} &= (\rho u, \rho u u + P, \rho u v, (E+P)u)^{\mathrm{T}} \,, \\ \mathbf{G} &= (\rho v, \rho u v, \rho v v + P, (E+P)v)^{\mathrm{T}} \,. \end{aligned}$$

where

$$P = (\gamma - 1)(E - \frac{1}{2}\rho \mathbf{U} \cdot \mathbf{U}) \, .$$

Initial Condition :



- Hugoniot-Rankine condition f
- Pre-Shock Temperature $T = $
- Pre-Shock Pressure $P = 0.5\ a$
- Xenon and Argon density a $10^{-3}\ \frac{g}{cm^3}$ and $\rho_{Ar} = 0.89 \times 10^{-}$ at half of the normal atmosph
- Specific heat ratio $\gamma = \frac{5}{3}$
- Atwood number $At = 5.4$
- Mach number $M = 4.46$
- Wave Length $\lambda = 3.6\ cm$
- Amplitude $a = 1.0\ cm$

Xenon-Argon interface definition :

$$S(x,y) = \exp(-\alpha|d|^{\beta}) \qquad 0 < d < 1$$

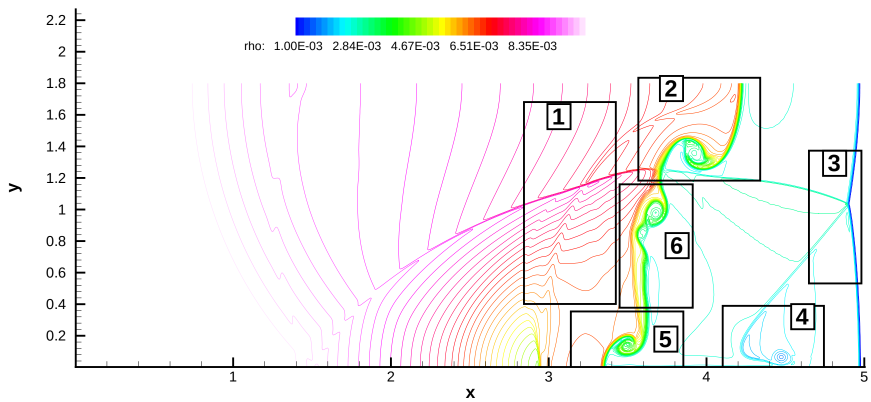
where

- $$d = \frac{(x_i + a\cos(2\pi y/\lambda) + \delta) - x}{2\delta}$$
- $\delta > 0$ is the interface thickness,
- $\beta = 8$ is the interface order,
- $\alpha = -\ln\epsilon$ with ϵ being the machine zero.

Algorithms :

- Spatial Algorithm :
  1. Combined Chebyshev and Fourier collocation method (Spectral),
     - Differentiation and Smoothing operations are done via an optimized (Costa & Don);
     - a 10'th and 9'th order exponential filter used for the differentiation and respectively.
  2. WENO fifth order finite difference scheme (WENO) with Lax-Frederick
  3. Symmetry property in y is utilized to reduce the cost of computation.
- Temporal Algorithm :
  Third order TVD Runge Kutta method (Shu and Osher).

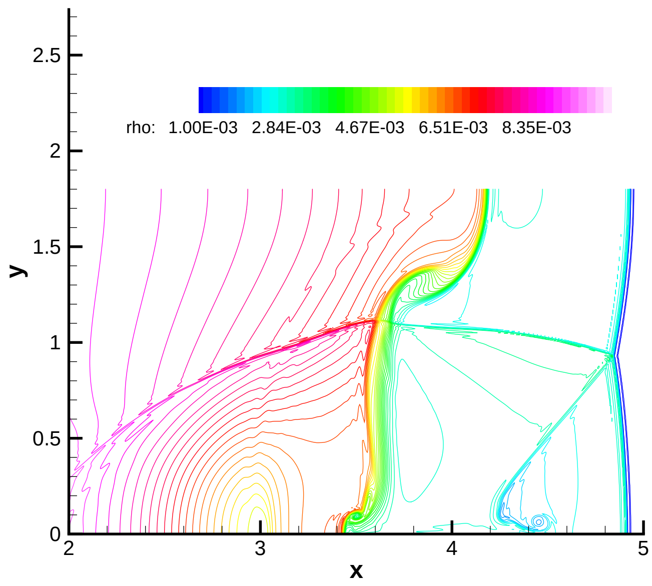
Regions of Interest :

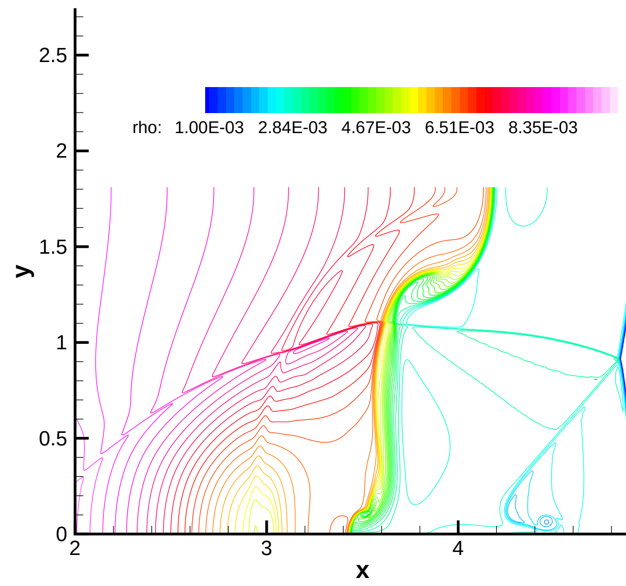


1. Reflected shock genera
   fraction;
2. The penetration of th
   (Ar) fluid forms the S
3. Triple point on the tra
4. A small jet and its vo
   Kelvin-Helmholtz inst
   stability along the con
   long time simulation;
5. The penetration of th
   (Xe) fluid forms the B
6. Vortical rollups of the

Convergence Study $(M = 4.46, \delta = 0.6\ cm, t = 50\ \mu s)$: Density

Spectral **WENO**

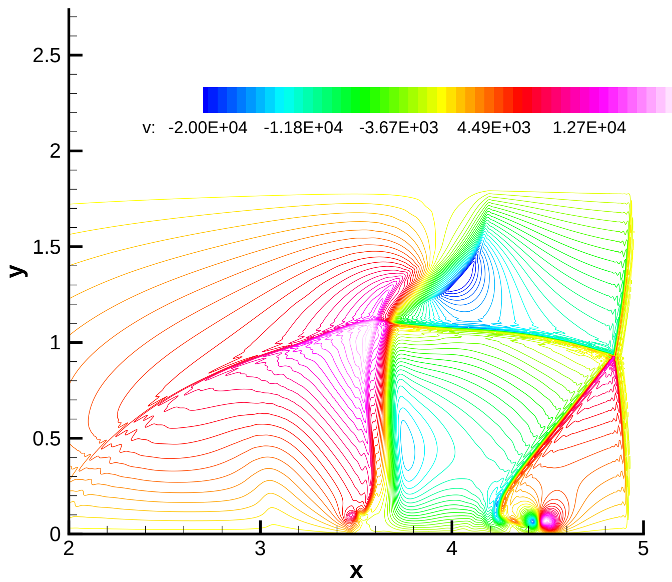


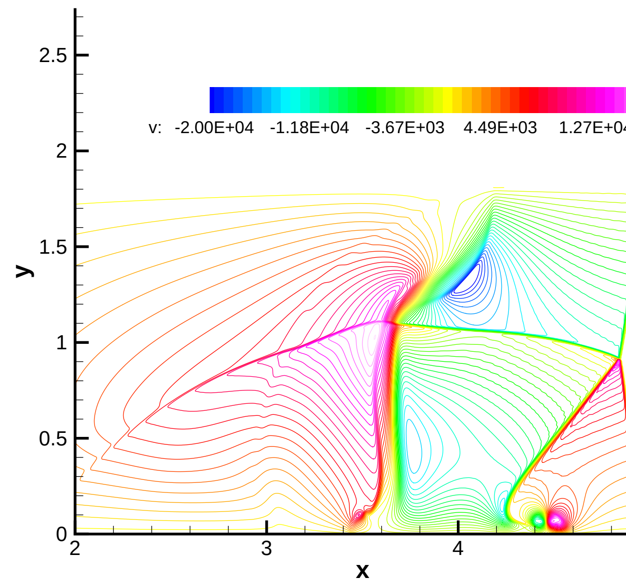


- Grid size for the Spectral and WENO schemes are 1024x512.

Convergence Study ($M = 4.46, \delta = 0.6\ cm, t = 50\ \mu s$) : V-Velocity

Spectral **WENO**

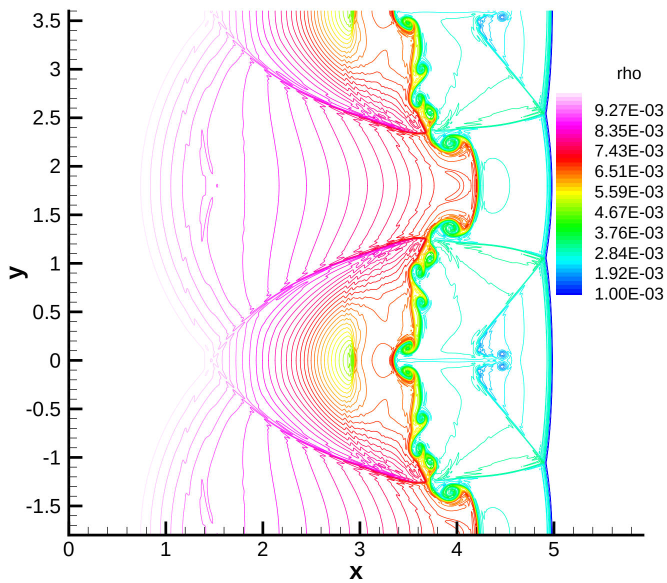


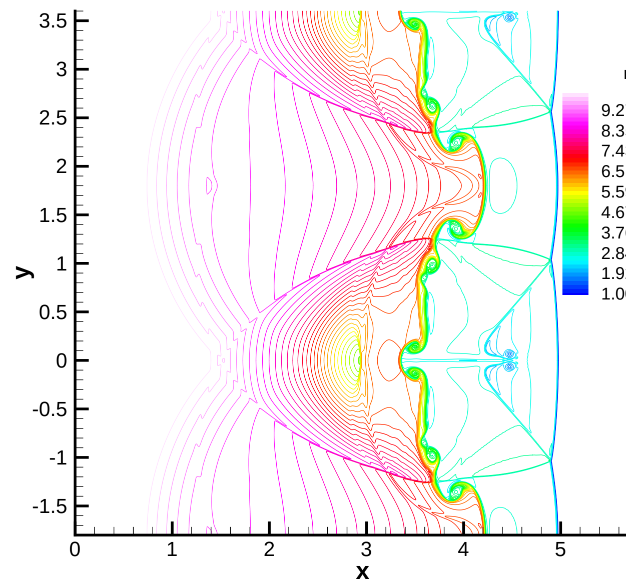


- Grid size for the Spectral and WENO schemes are 1024x512.

Convergence Study ($M = 4.46, \delta = 0.2\ cm, t = 50\ \mu s$) : Density

Spectral **WENO**

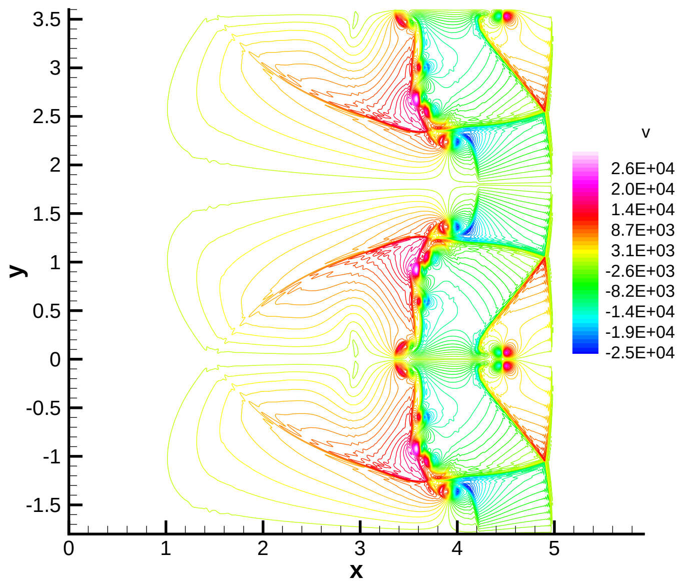




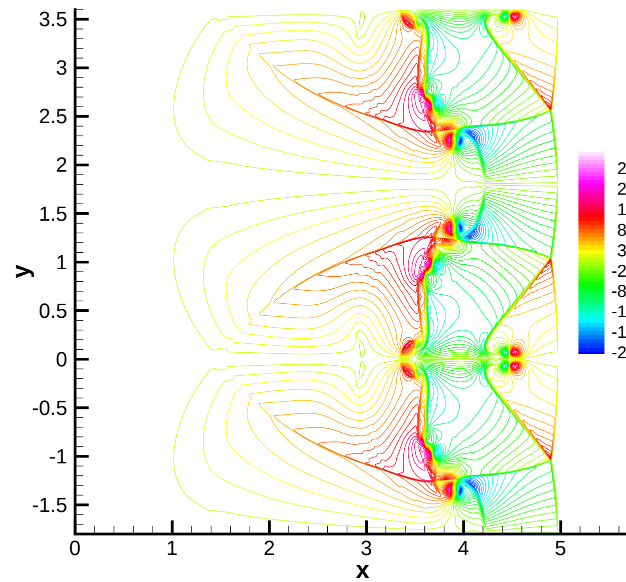
- Grid size for the Spectral and WENO schemes are 1024x256 and 1024x512

Convergence Study ($M = 4.46, \delta = 0.2\ cm, t = 50\ \mu s$) : V-Velocity

Spectral

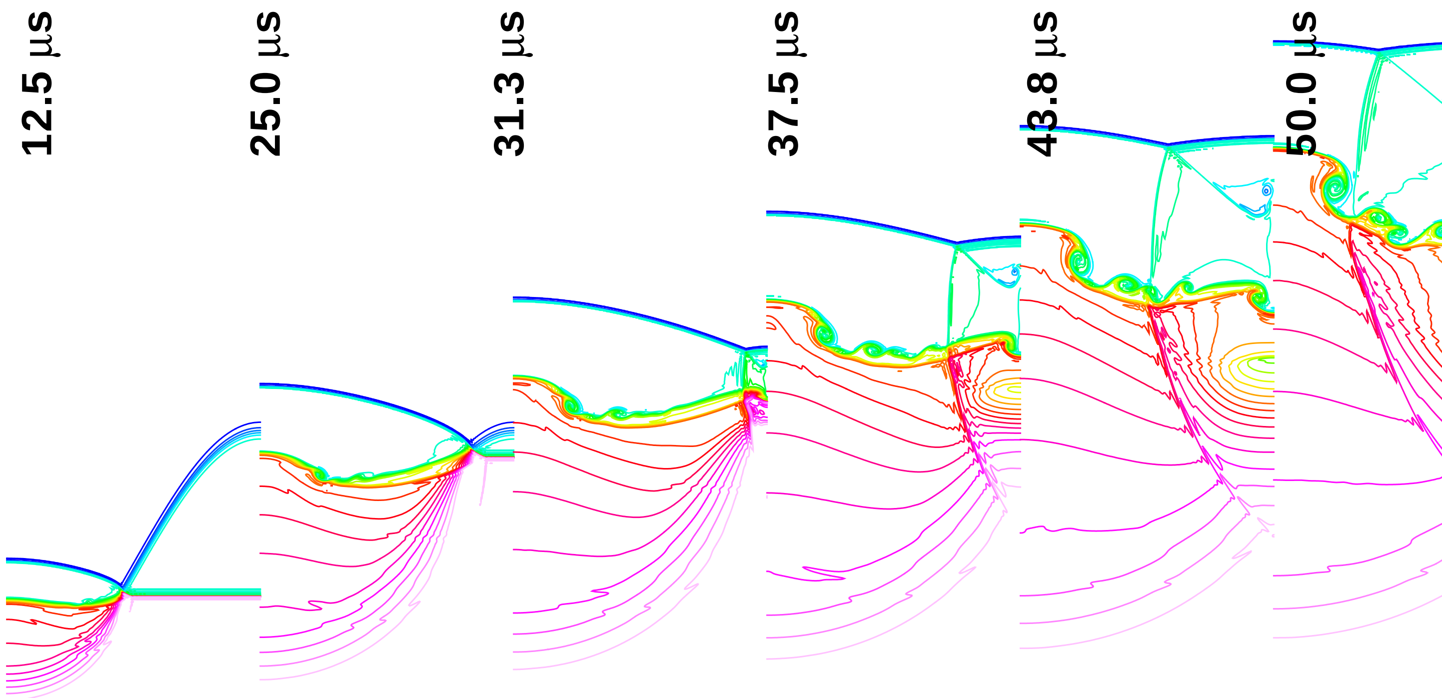


WENO

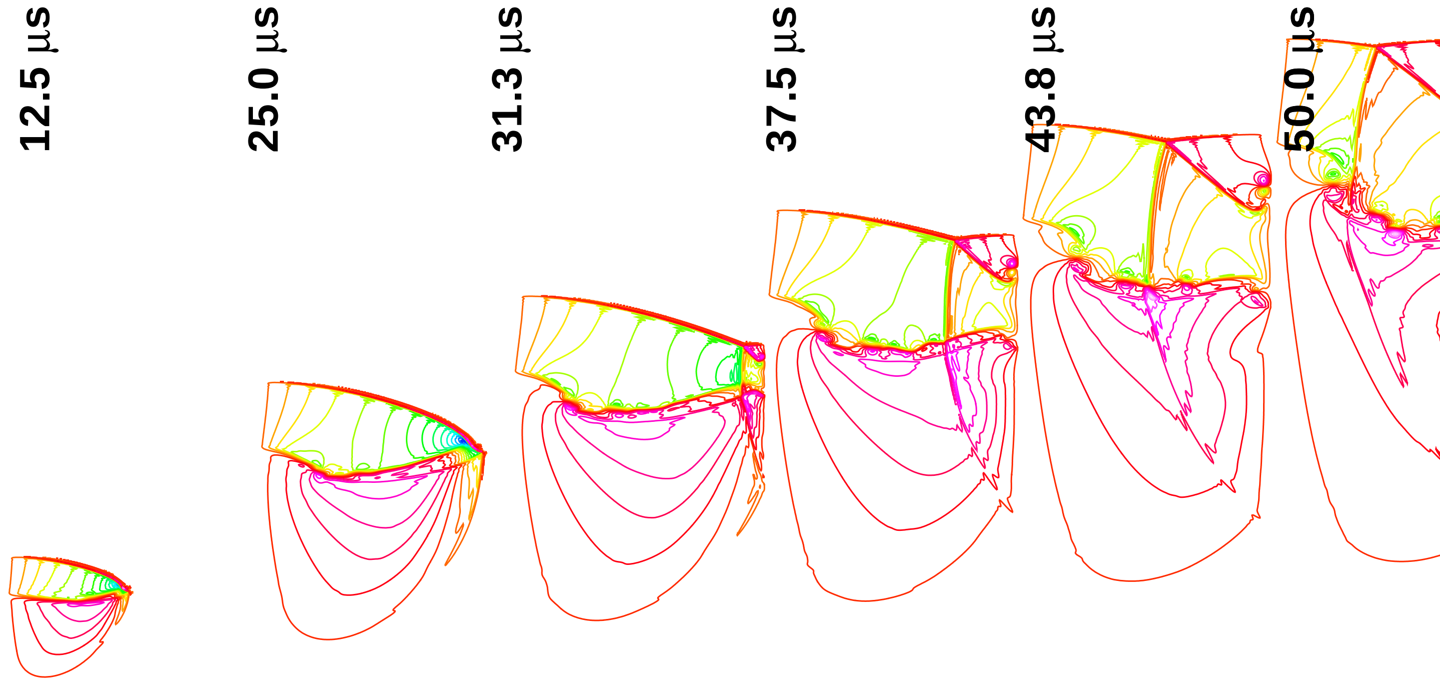


- Grid size for the Spectral and WENO schemes are 1024x256 and 1024x512

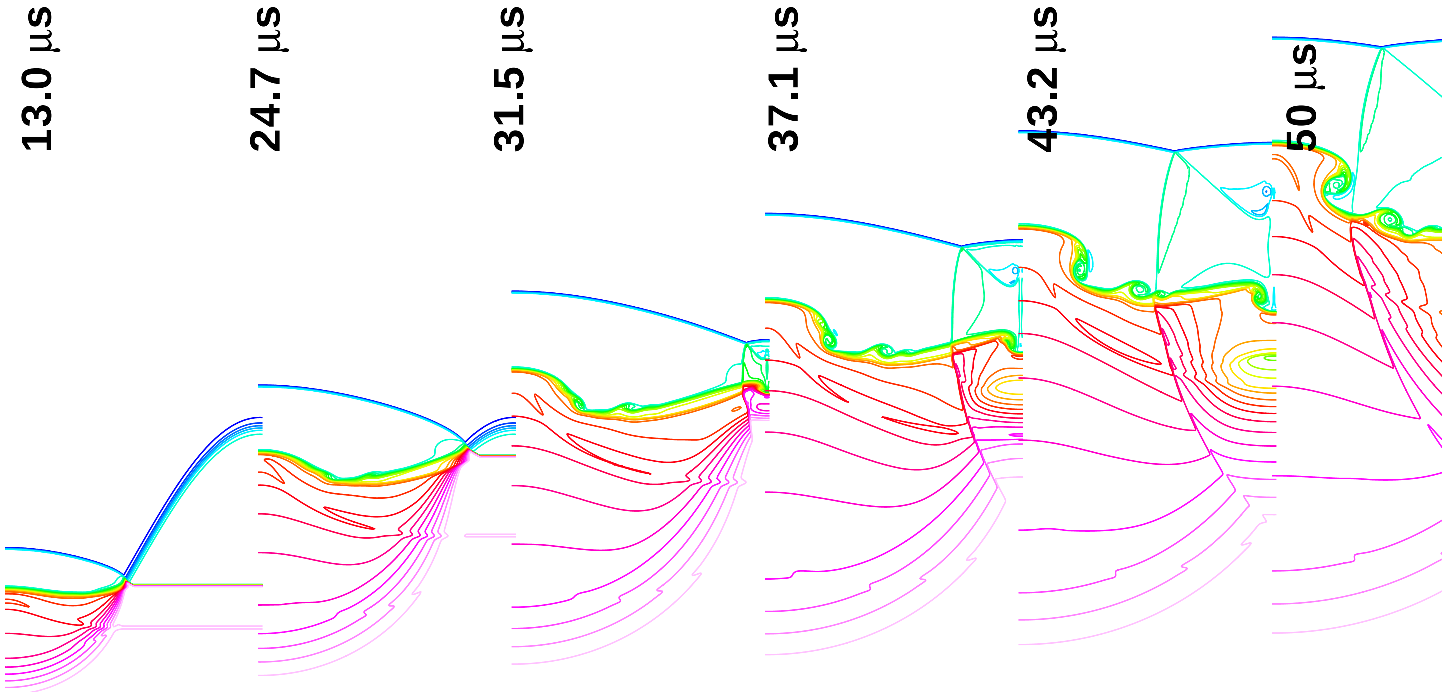
Snapshot of the Density ($M = 4.46, \delta = 0.2\ cm$) : Spectral Scheme



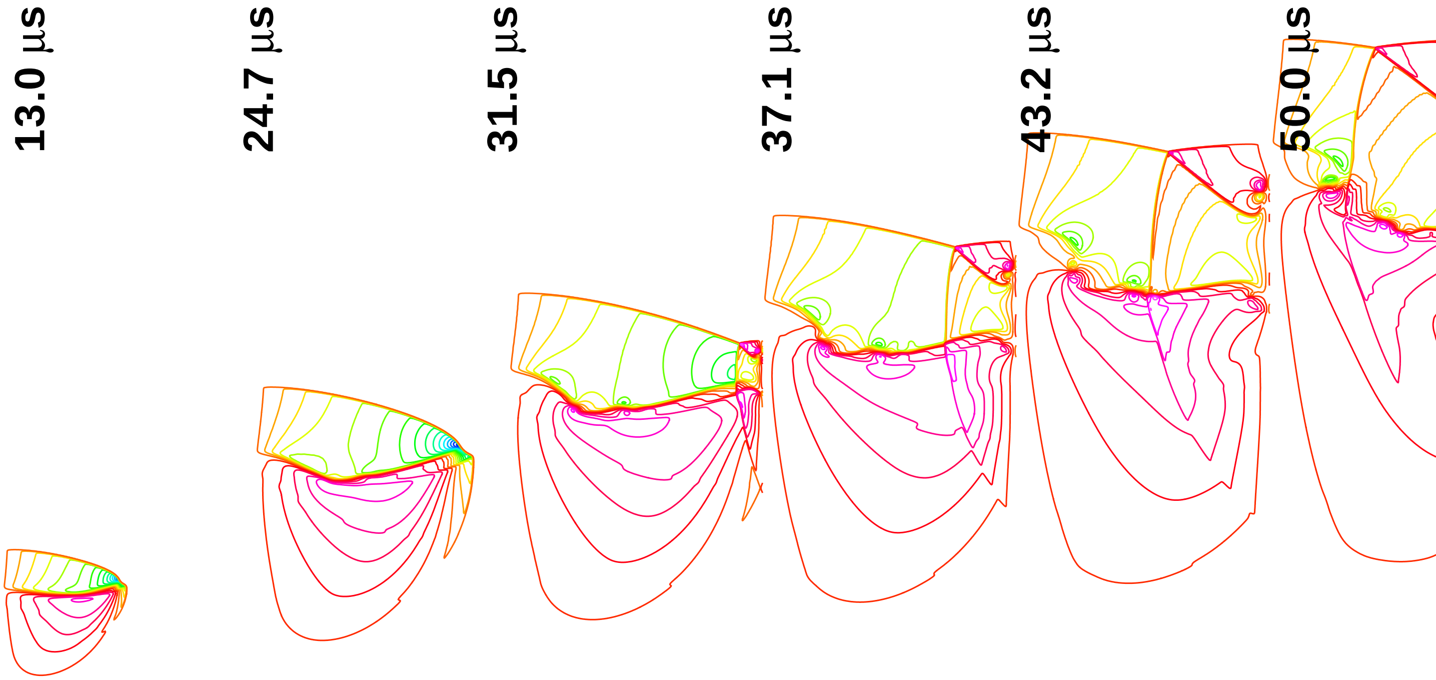
Snapshot of the V-Velocity ($M = 4.46, \delta = 0.2\ cm$) : Spectral Sche



Snapshot of the Density ($M = 4.46, \delta = 0.2\ cm$) : WENO Scheme



Snapshot of the V-Velocity ($M = 4.46, \delta = 0.2\ cm$) : WENO Schem

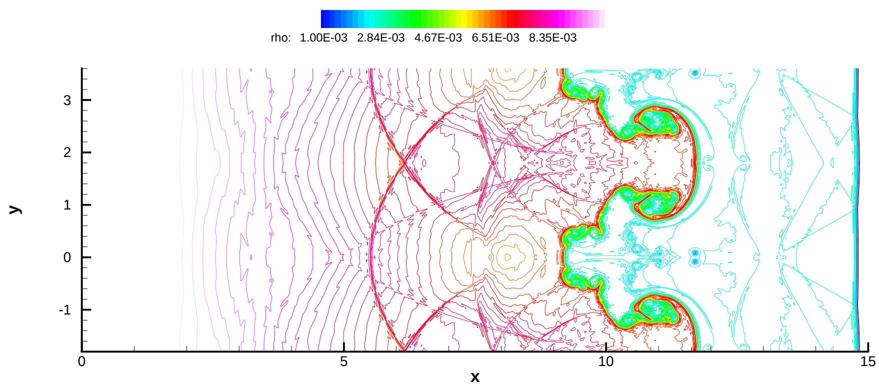


Observations :

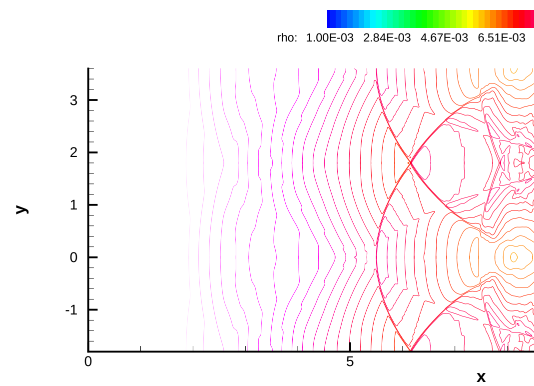
- Good agreement of the Global large and medium features between the Spe WENO scheme.
- Some discrepancy of the fine scale vortical structures along the gaseous inte simulations of this sensitive nature to small perturbation (physically and/

Long time Case ($M = 4.46, \delta = 0.2\ cm, t = 124\ \mu s$) : Density

Spectral

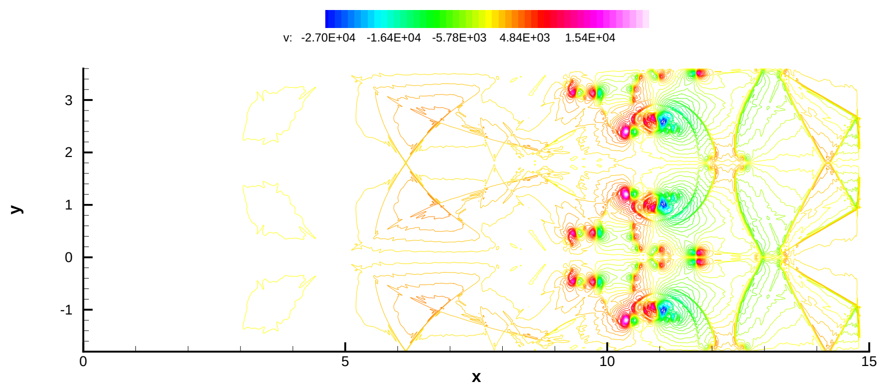


WE

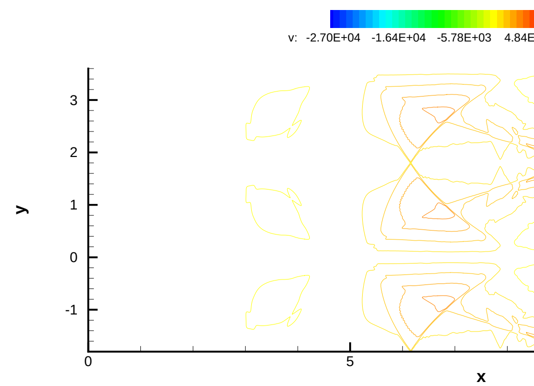


Long time Case ($M = 4.46, \delta = 0.2\ cm, t = 124\ \mu s$) : V-Velocity

Spectral



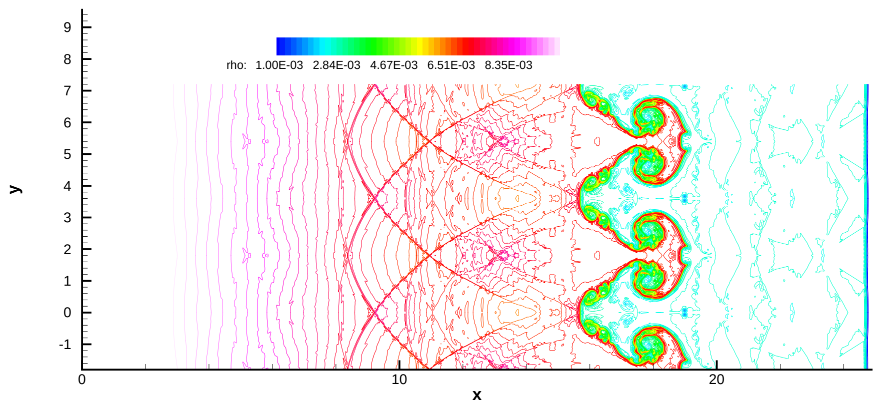
WE



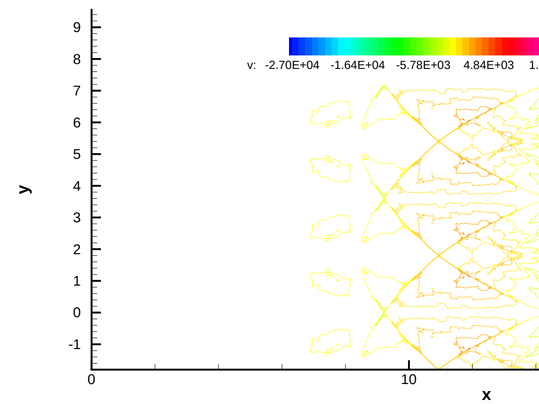
Richtmyer-Meshkov Instability (Cont.)

Large Domain Case ($M = 4.46, \delta = 0.2\ cm, t = 237\ \mu s$) : Spectral sc

Density



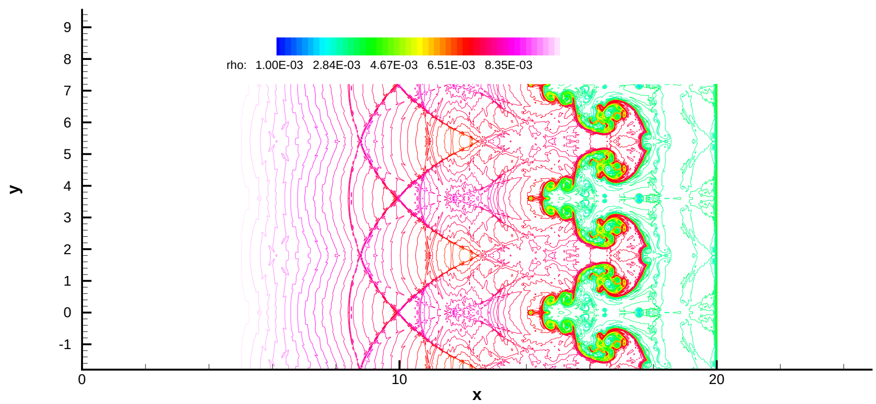
V-Ve



Richtmyer-Meshkov Instability (Cont.)

High Mach Number ($M = 8, \delta = 0.2\ cm, t = 200\ \mu s$) : Spectral sche

Density



V-Ve