



# Pacific Institute for the Mathematical Sciences

## New PIMS Director: Alejandro Adem

Alejandro Adem is the new Director of PIMS. His five-year term will begin on July 1, 2008.

Dr. Adem brings extensive administrative experience to PIMS. He served as Chair of the Department of Mathematics at the University of Wisconsin–Madison between 1999 and 2002. Since 2005, he has been the Deputy Director at PIMS. Dr. Adem has outstanding credentials as a scientific organizer. He served for four years as Co–Chair of the Scientific Advisory Committee at the Mathematical Sciences Research Institute at Berkeley (MSRI), and as member of the MSRI Board of Trustees. Since 2005, he has been a member of the Scientific Advisory Board for the Banff International Research Station (BIRS).

“The process of looking for the next PIMS Director started with a call for applications in January, 2007,” said Ivar Ekeland, current PIMS Director. “I am very grateful to the Search Committee to have come up with such an outstanding candidate, and for UBC to have made this appointment possible. This is excellent news for PIMS and I am looking forward to working with my successor, both before and after July 1.”

Dr. Adem said, “It is a great honour for me to assume the position of Director of PIMS. Under the leadership of Ivar Ekeland, PIMS has developed into a world–class centre for mathematical research, industrial collaboration and educational outreach. The recent designation of PIMS as a Unité Mixte Internationale of the French CNRS



Alejandro Adem

continued on page 4

### Inside this issue

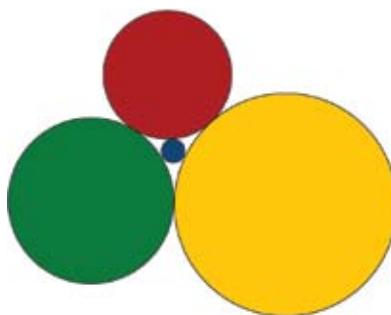
<i>Director's Message</i>	2
<i>PIMS' News</i>	3
<i>CNRS Agreement</i>	5
<i>CRM-Fields-PIMS Prize 2008</i>	7
<i>PIMS' First Decade</i>	12
<i>Interview with Steve Smale</i>	14
<i>Equidistribution and Primes by Peter Sarnak</i>	15
<i>New PIMS Board, SRP Members</i>	22
<i>PIMS Scientists Receive Honours</i>	24
<i>PIMS' Conferences</i>	26
<i>New PIMS CRGs</i>	32
<i>Upcoming PIMS Events</i>	33
<i>Education News</i>	34

### University of Regina Joins PIMS

The Pacific Institute for the Mathematical Sciences (PIMS) welcomes the University of Regina as a full member, effective December 1, 2007.

Read more on the new PIMS member university on page 3.

**Special to PIMS:**  
**Equidistribution and Primes by Peter Sarnak (Princeton University & Institute for Advanced Study Princeton, New Jersey)**



Read the article on page 15

### PIMS becomes CNRS Unité Mixte Internationale

The Pacific Institute for the Mathematical Sciences has become an Unité Mixte Internationale of the French Centre National de la Recherche Scientifique (CNRS). The agreement has been signed by the six PIMS member universities (Simon Fraser University, the University of Alberta, the University of British Columbia, the University of Calgary, the University of Victoria and the University of Washington), effective September 2007. PIMS is now an integral part of the CNRS system, and will host CNRS researchers on a regular basis and have closer ties with the French scientific community.

CNRS recognition is a label of quality. There are only four UMIs in mathematics around the world: in Moscow, Rio, Santiago, Vienna, and now PIMS in Western Canada and the Pacific Northwest.

story continued on page 5

# Director's Message

by Ivar Ekeland

I will say it one last time: PIMS is a unique organization. Up to now, the standard model for math institutes has been the Princeton one: institutes were supposed to be places that host distinguished visitors and thematic programs: to benefit from them, you have to spend time at the institute. On the contrary, you do not have to go to PIMS: PIMS comes to you. At each of PIMS' seven member universities, there is a PIMS site office, with a PIMS director and an administrative assistant, who together are in charge of organizing PIMS events on site and bringing visitors and post-doctoral fellows to their university.

The PIMS model has weaknesses. First of all, it is unfamiliar, because it is new. We do not yet have the track record of the older institutes, and much of the work we do, such as bringing the mathematical community together across a vast geographical region, goes unnoticed, because people do not expect it. To counter that effect, we must do a better job of promoting ourselves, and explaining to the world who we are and what we do. A second weakness comes from the difficulty of coordinating seven different sites, each of which has to serve its own university, but also the greater mathematical community. This we do through programs that bring together the whole PIMS community, such as the Collaborative Research Groups, our educational programs with the First Nations, and the Industrial Problem-Solving Workshops. This also requires us to find creative ways of weaving together the administrative staff and processes across the different sites so that they function as a single organization.

The strengths of the PIMS model, on the other hand, are overwhelming. They never were more apparent than on January 22, 2007, where over one hundred scientists from all over the world showed up for the PIMS NSERC site visit. I think all of those who were present that day will remember it. The amount of support we received that day, not only from mathematicians, but also from other scien-

tists, from the universities and from local communities, was incredible. The day ended with a presentation from the CRGs (five minutes each!) which was a real firework, showing mathematics at its best, in terms of eternal quality and relevance to contemporary issues.

This happened because PIMS has been able to unleash and leverage the scientific potential in the various sites. The role of the site directors in this cannot be overstated: it is for them to bring PIMS to their university, by encouraging initiatives across the whole spectrum of mathematics, and bringing to their colleagues' attention the full range of possibilities that is open to them within PIMS: post-doctoral fellowships, individual events, distinguished visitors, CRGs, and our international partnerships. The success of PIMS is largely theirs.

We are happy to see that the success of the PIMS model is attracting ever more scientists and institutions who want to be part of this unique endeavour. It gives me great pleasure of welcoming the University of Regina as a full member, and the Camosun College as the first PIMS Education Associate. I am also proud that PIMS has been recognized by CNRS as an Unité Mixte Internationale, an honour bestowed to only four other mathematical institutes in the world. This opens new avenues for the PIMS community, and is rich in promises for the future.

I would like to end this message by welcoming Alejandro Adem, who will be taking over as PIMS Director on July 1. He has already contributed enormously to PIMS as Deputy Director, together with the site directors, the Board, the Scientific Review Panel, and all those, staff, scientists, educators, private persons, who are giving part of their time and energy to this unique institution. It has expanded in the past five years, going from five universities to seven, affiliating with the French CNRS, and creating a vast exchange network around Latin America and the Pacific Rim. At this time when so many would have us believe that mankind is breaking down into separate cultures, let us show the world that science can create a vibrant community across five continents.



(l to r) Ivar Ekeland, PIMS Director, and Arvind Gupta, Director of MITACS.

## Camosun College becomes first PIMS Education Associate

By David Leeming (U.Victoria)

In February 2008, the Pacific Institute for the Mathematical Sciences entered into an exciting new partnership with Camosun College in Victoria with PIMS and Camosun signing a Memorandum of Understanding (MOU) that makes Camosun College the first PIMS Education Associate. The agreement is for three years and is renewable. For an annual fee, Camosun will have access to the educational resources of PIMS. In addition, they will be able to sponsor jointly with PIMS one educational outreach activity per year. Camosun College Mathematics Department will soon appoint one of their faculty to be the PIMS liaison.

Grant MacEwan College in Edmonton is expected to sign a MOU shortly to become the first Alberta college to become a PIMS Education Associate. It is expected that several other B.C. and Alberta colleges will become PIMS Education Associates in the next few months.



# PIMS News

## University of Regina joins PIMS

The Pacific Institute for the Mathematical Sciences (PIMS) welcomes the University of Regina as a full member, effective December 1, 2007.

“Many disciplines rely on mathematics and statistics even at the elementary level,” said Shaun Fallat, Associate Professor of Mathematics and PIMS Site Director at U.Regina. “For example, biologists draw conclusions from their data sets by utilizing many standard statistical methods. However, they rarely discuss new and innovative ideas or techniques with statisticians. Mathematicians are always in search of new and interesting problems and many such problems come from the applied sciences and the like. PIMS can bring these groups together, partly because they are interested in funding such collaborations, via postdoctoral opportunities, lecture series, conferences, and summer schools.”

“PIMS prides itself on being a campus-wide initiative that involves the mathematical sciences in some manner, and since nearly every subject makes use of mathematics at some level, such a mandate is a natural and worthwhile one.”

Faculty and researchers in the Department of Mathematics and Statistics at U.Regina have an outstanding record for their research strength in key areas. The field of linear algebra is one example, where the department has established researchers in core linear algebra, numerical linear algebra, and operator theory. A significant amount of research at U.Regina is in areas of applied mathematics, highlighted by the expertise acquired by researchers in the Prairie Drought Project (led by P. Leavitt, now a Tier I CRC), which can be brought to current PIMS collaborative projects.

U.Regina researchers have experience collaborating with industry partners. This strength will be leveraged in 2008 when U.Regina hosts the 11<sup>th</sup> PIMS Graduate Industrial Mathematics Modelling Camp and the 12<sup>th</sup> PIMS Industrial Problem Solving Workshop, held June 9-20, 2008 in Regina (<http://www.pims.math.ca/ipsw/>)

U.Regina’s mathematical scientists have a long-standing tradition of being actively involved in a variety of outreach initiatives at all levels: locally (problem solving workshops), provincially (math camps and challenges), nationally (editorships in *Crux Mathematicorum*), and internationally (Math Central). U.Regina faculty members are active in aboriginal mathematics education and outreach, attracting significant funding (NSERC CRYSTAL grant) and hosting an international conference (Dreamcatching 2007). Faculty at U.Regina are key contributors to the development of aboriginal mathematics education at the provincial and national levels.

Dr. Fallat ([sfallat@math.uregina.ca](mailto:sfallat@math.uregina.ca)) is the PIMS site director at U.Regina, with administrative support provided by Laurie Cosgrove. Katherine Bergman, Dean of Science, joins the PIMS Board of Directors as the U.Regina representative.

U.Regina joined PIMS as an affiliate member in 2005. With the addition of U.Regina as a full member, PIMS now has member universities in the three westernmost Canadian provinces, as well as Washington State.



Dr. John Archer Library at U.Regina

## PIMS NSERC Renewal: 2008-2013

The Pacific Institute for the Mathematical Sciences is pleased to announce that its Natural Sciences and Engineering Research Council of Canada (NSERC) grant has been renewed for a further five-year period, and increased to \$5.5-million for the period 2008-2013.

Ivar Ekeland, PIMS Director, said, “This has been a great year for PIMS. Alberta doubled the size of our grant, the French CNRS is entering into a permanent agreement, the only one of its kind in North America, British Columbia is interested in our First Nations Education program, and now we have this great news from NSERC. This shows the extent of support for PIMS, both inside and outside our borders. We are all tremendously encouraged, and are committed to live up to these expectations.”

With continued funding secured from a variety of sources, PIMS will continue to address its core mission, as outlined in its statutes: to promote



**NSERC  
CRSNG**

continued on page 4



# The Changing Face of PIMS

## New PIMS Director: Alejandro Adem

continued from page 1

is a recognition of PIMS' high standing in the international community. I am extremely grateful to all of my colleagues in the PIMS community for their support and look forward to working with them to further develop PIMS as one of the leading mathematical institutes in the world."

Presently the PIMS Deputy Director, Dr. Adem is a Professor of Mathematics at the University of British Columbia, and holds the Canada Research Chair in Algebraic Topology. He received his B.Sc. in 1982 from the National University of Mexico, and his Ph.D. from Princeton University in 1986. After holding a Szegő Assistant Professorship at Stanford University and spending a year at the Institute for Advanced Study in Princeton, he joined the faculty of the University of Wisconsin–Madison in 1990, where he remained until 2004. Dr. Adem has held visiting positions at the ETH–Zurich, the Max Planck Institute in Bonn, the University of Paris VII and XIII, and at Princeton.

Dr. Adem's mathematical interests span a variety of topics in algebraic topology, group cohomology and related areas. He has written numerous research articles as well as two highly regarded monographs. He has given over 200 invited talks on his research throughout the world, and has supervised several Ph.D. students and postdoctoral fellows. He was awarded the U.S. National Science Foundation Young Investigator Award in 1992, a Romnes Faculty Fellowship in 1995 and a Vilas Associate Award in 2003. He has extensive editorial experience and is currently an editor for the *Memoirs and Transactions* of the American Mathematical Society.

Dr. Adem brings a wealth of experience in organizing international collaborations, connecting Canadian mathematical scientists with colleagues abroad. This includes his leadership in organizing the first joint meeting between the Canadian and Mexican mathematical societies in 2006, as well as his crucial role in the development of the Pacific Rim Mathematical Association (PRIMA). He will continue to build on PIMS' outstanding record of mathematical collaboration between academic, industrial and international partners.

## PIMS NSERC Renewal: 2008-2013

continued from page 3

research in the mathematical sciences; to strengthen ties and collaboration between the mathematical scientists in the academic community, in the industrial and business sector, and in government; to enhance education and training in mathematical sciences, and broadening communication of mathematical ideas; and to create strong mathematical partnerships and links within Canada and with organizations in other countries.

PIMS will immediately implement the new ideas contained in its successful proposal: in September, 2007, PIMS inaugurated the first of its International Graduate Training Centres (IGTC) in Mathematical Biology, bringing together disciplines and colleagues from PIMS member universities in Western Canada and the Pacific Northwest, and beyond. PIMS will also continue its successful post-doctoral fellowships program and will launch new collaborative research groups (CRGs) and industrial initiatives in strategic areas, such as the mathematics of the environment, in close cooperation with our international partners in Latin American and around the Pacific Rim.

PIMS is grateful to NSERC for the increase in support for the mathematical institutes in Canada, and congratulates the Fields Institute and the Centre de recherches mathématiques (CRM) on the success of their applications.

PIMS thanks all members of the community for their support during the renewal process and the site visit, and their outstanding contributions to science during the past granting period, which has made PIMS' continued success possible.

More details about PIMS' exciting new IGTC program can be found on page 6

## Ian Putman, new U.Victoria Site Director



Ian Putman is the new PIMS Site Director at the University of Victoria. His term began on January 1, 2008.

Dr. Putman is a Professor in the Department of Mathematics and Statistics at the University of Victoria. He is the Canada Research Chair in Operator Algebras and Dynamical Systems, and a Fellow of the Royal Society of Canada.

Dr. Putman is recognized worldwide as a top expert on the interrelation of Operator Algebras and Dynamical Systems. His main area of research interest is in the interaction between  $C^*$ -algebras and topological dynamics.

Dr. Putman replaces Christopher Bose, who was the U.Victoria PIMS Site Director from July, 2004, to December, 2007.

## Gerald Cliff awarded second term as U.Alberta Site Director

Gerald Cliff will stand for a second term as PIMS Site Director at the University of Alberta.

Dr. Cliff is a Professor in the Department of Mathematical and Statistical Sciences at U.Alberta. He obtained his B.Sc. at McGill in 1970 and his Ph.D. in Mathematics at the University of Illinois at Urbana in 1975. He has been at the University of Alberta since 1975.

His main areas of interest is representation theory of groups, as well as integral and modular representations of finite and algebraic groups, and also quantum groups.

Dr. Cliff first became PIMS Site Director at U.Alberta in August, 2004.



# PIMS becomes CNRS Unité Mixte Internationale

continued from page 1

CNRS has committed to send to PIMS two or more *chargés de recherches* or *maîtres de conférences*. These researchers will spend one year at a PIMS site, working with the PIMS research teams. As of September 2007, there are three of these individuals at PIMS Universities: two at UBC (Messrs Druet and Vovelle) and one at U. Washington (Mr. Ge), and they will stay until summer 2008, participating in the life of the university. The choice was made on a competitive basis, and PIMS is at present opening the competition for the 2008-09 academic year.

PIMS has committed to give two Postdoctoral Fellowships a year to French applicants, as a matter of reciprocity. As of September 2007, there are two recipients, one at SFU and one at U. Alberta.

This program has just begun, but it already has a tremendous success: clearly mathematicians in B.C. and in France are eager to develop ties and to work together on PIMS programs such as Probability and Statistics, Mathematics of the Environment, or Mathematical Biology. PIMS is holding a thematic summer program on “Economics and Finance of Sustainable development” at UBC in the summer of 2008, with the support of CNRS and of the Chaire de Développement Durable (Polytechnique–EDF–Dauphine). In the long run, PIMS plans to expand that into a permanent scientific program, involving PIMS and the CNRS.

## What is the CNRS agreement ?

by Ivar Ekeland, PIMS Director

The Centre National de la Recherche Scientifique (CNRS) is the French national research agency for science and humanities. It is a large organization: 30,000 employees (staff and researchers) and a budget of three billion euros. It is a public agency, so that all employees are civil servants (and hence tenured).

CNRS operates in two ways. It has its own centres (the so-called *laboratoires propres*), notably in experimental sciences, which require considerable amount of equipment. But it also associates with universities to create joint centres (the so-called *unités mixtes de recherche*, UMR). In that case, the CNRS and the university are bound by a four-year contract, which specifies the commitments of each partner; this contract is renewable. Typically, the university commits to provide space and research time from its faculty, and the CNRS commits to provide personnel. Equipment and funds are shared.

The UMR is the standard way CNRS operates in mathematics. Every CNRS researcher has to belong to some UMI. On the other hand, not all faculty belong to an UMI: some universities do not have any, and even in universities who have one, not all faculty at the university are allowed to participate. Having an UMI is a label of quality for the university, and belonging to an UMI is a label of quality for the professor. In this way, the CNRS operates as an evaluation agency throughout the French university system.

CNRS has concluded a similar agreement with a few select universities throughout the world, four exactly: IMPA in Rio de Janeiro, CMM in Santiago, the free University in Moscow, and the Wolfgang Pauli Institute in Vienna. These are called *Unités Mixtes Internationales*, UMIs, and PIMS is now the fifth one. The agreement signed with the six PIMS universities (and which will extend to new members as they come in) is very similar to the standard UMR agreement: PIMS provides space and working conditions, while CNRS provides personnel, namely researchers.

In practice, this means that CNRS commits to send to PIMS every year at least two CNRS researchers. They will spend one year at a PIMS



university, keeping their salary from France (usually, they are entitled to some supplemental income from CNRS as well) and the associated benefits (welfare, pension). The host university commits to providing them with office space, equipment and good working conditions, and to treat them as visiting faculty. This, of course, is tremendously beneficial to the university, which gets these top-notch researchers essentially for free.

What is in it for CNRS ? They feel that their young researchers (*chargés de recherches*, as opposed to the more senior *directeurs de recherches*) will gain valuable experience at PIMS. The leadership of CNRS has been particularly impressed by the distributed structure of PIMS and its international connections. They also like the role we have played in opening up new areas of mathematics, which are less represented in France, such as environmental statistics, climate studies, and the management of natural resources.

Last but not least, the CNRS agreement extends to university personnel by analytic continuation. To be precise, faculty (*professeurs* or *maître de conférences*) can be seconded by their home university to the CNRS, which in turn can second them to PIMS. The net result is the same: they retain their university salary, plus whatever they are able to negotiate from CNRS, and come to PIMS for one year. The total number of such personnel is not fixed, but I anticipate it to be two or more.

In 2007-08, we had three researchers coming from France on the CNRS agreement: two *chargés de recherches* at UBC and one *maître de conférences* at UW. In 2008-09 we are expecting seven such researchers: three at U. Calgary, one at U. Victoria, three at UBC. This is only a beginning. I am sure that as the agreement falls into place and the universities realize its potential, many ambitious research projects will flourish between PIMS and the French scientific community. PIMS will become a hub between two worlds, the Pacific Rim on one side and Europe on the other.



## IGTC in Mathematical Biology Receives \$120,000 From B.C. Government

The British Columbia Ministry of Advanced Education will support the PIMS International Graduate Training Centre (IGTC) in Mathematical Biology with a \$120,000 grant to finance three, two-year graduate student fellowships.

The IGTC in Mathematical Biology, opened in 2007, provides a specialized training program for students in mathematical biology. This subject is one of strategic importance in Western Canada due to its direct connection to mathematical ecology, genomics, bioinformatics, systems biology, and biophysics, to name only a few subject areas. Important local industries, such as those of forestry and fisheries, depend on fragile ecosystems that are being modeled with increasing accuracy and sophistication by mathematical biologists. Training the future specialists in this interdisciplinary field is of major importance to both industry and academia in British Columbia.

## PIMS New Program: International Graduate Training Centres (IGTC)

The PIMS International Graduate Training Centres emerged as one of the new initiatives in the new PIMS NSERC granting period, 2008-2013. It is the goal of the IGTCs to train the next generation of researchers, focusing on graduate students, to apply new and traditional mathematics to the major scientific problems of our time.

Each IGTC will develop a specialized graduate program at one or several PIMS universities. PIMS will serve as a catalyst for the programs, by identifying the appropriate areas of mathematics; by supporting summer schools and conferences; by bringing distinguished scholars to the PIMS universities to teach and share their knowledge with IGTC students; and by awarding graduate fellowships to students in the program.

The IGTCs will turn PIMS universities into international hubs for graduate studies in emerging and strategic areas of mathematics. Each IGTC will be supported by, and operate closely with, strong research groups, so that students and researchers from around the world will view the IGTC universities as a destination for the exploration of these new areas of mathematics. PIMS expects that within the five-year mandate, the successful IGTCs will become a worldwide centre of research, learning and training.

The first PIMS IGTC is in the area of Mathematical Biology. Opened in September, 2007, the IGTC will focus on the training of graduate students in mathematical biology at PIMS universities, centred at U. Alberta and UBC, while bringing together students and researchers from across all PIMS universities.

[http://www.pims.math.ca/igtc/math\\_biology/](http://www.pims.math.ca/igtc/math_biology/)

## PIMS' First Nations Teacher Training Program Receives B.C. Support

The British Columbia Provincial Government is providing \$130,000 to PIMS to help teachers increase math participation of aboriginal students in First Nations schools.

"This funding will support a partnership to help aboriginal students build strong math foundations from kindergarten to Grade 12, reducing the barriers to success at the post-secondary education level," said Advanced Education Minister Murray Coell. "We are creating a supportive education path that will lead to higher learning, and partnerships like this increase participation by making math more relevant and accessible to aboriginal students."

PIMS has created programs in consultation with First Nations schools and the First Nations Education Steering Committee (FNESC) to provide teachers with training and assistance to help students learn math. The program includes a website for remote schools where teachers can exchange ideas, outreach events, and forums to bring mathematicians and First Nations educators together and mentoring by students at local universities.

"The support given to PIMS by the Ministry of Advanced Education will lead to a significant expansion of the scope and impact of our partnership with First Nations schools," said Alejandro Adem, PIMS deputy director. "By tackling the mathematical disadvantages that their students face early on, we seek to nurture the development of a critically important pipeline of Aboriginal undergraduate students in subjects such as science, engineering and computer science."

The program is being expanded to four more First Nations schools in Lytton, Barriere, Bella Bella and Port Alberni, following a successful pilot in 2005 at the Sk'elep School of Excellence in Kamloops. PIMS tested students in grades 3 to 6 at the beginning of the pilot program to determine basic skill levels. The preliminary test results showed that in one year of the program, mastery of addition and subtraction, multiplication and division, and problem solving improved by an average of over 50 per cent.

"During the pilot program at the Sk'elep School of Excellence in Kamloops, students became more comfortable with basic math skills and were encouraged to take more math courses," said Kamloops-North Thompson MLA Kevin Krueger. "It's important that we give First Nations students the tools they need to pursue post-secondary education, and I'm thrilled to see this program being expanded to include Barriere."

"By leaving behind the philosophy of reduced expectations, introducing new and exciting ways to teach mathematics in schools, and promoting good role models for students and teachers, we feel that PIMS can make a significant difference to the way First Nations students view science and technology, and prepare them for a better future in society," said Ivar Ekeland, PIMS Director. "We are happy that the B.C. Government shares this vision, and gives us the opportunity to turn this vision into a reality."

The creation of the pilot teacher training and mentorship program was made possible by generous gifts from Ken Spencer, Andrew Wright, Haig Farris and the Gabriella Rosenbaum Foundation.



# SAGE Wins International Open-Source Software Competition

Open-source mathematical computing software SAGE (Software for Algebra and Geometry Experimentation), led by William Stein (University of Washington), has won first prize in the scientific software category of the 2007 Trophées du Libre, an international free software contest.

SAGE, an alternative to Magma, Maple, Mathematica and MATLAB, was first conceptualized by Dr. Stein in 2005, while he was at Harvard University. Commercially available mathematical computing software traditionally charge high licensing fees, and due to the commercial closed-source nature of the software, mathematicians cannot examine the calculations behind the computed answer.

From a compilation of other open-source mathematical software, and writing in the gaps in the code, Dr. Stein led the project to create the SAGE prototype over a year and a half. The open-source nature of the software means that any interested developer can look over the code, and offer suggestions for improvements.

Students make up a majority of SAGE users and developers – comprising about 60 percent of developers and about 70 percent of users. New versions are constantly being released as the software is updated, based upon the work of hundreds of volunteer developers.

In addition to on-line collaborations and work done at U. Washington, regularly scheduled “SAGE Days” are held, bringing together SAGE volunteer developers in one place for discussion and lectures. The first SAGE Day was held at the University of California at San Diego in February, 2006, and subsequent SAGE Days take place across the United States and in the United Kingdom.

PIMS is co-sponsoring SAGE Days 9, August 8-16, 2008, to be held at Simon Fraser University (<http://www.pims.math.ca/science/2008/08mgvw>). SAGE Days 9 will be the first SAGE event hosted in Canada.

For more information on SAGE, visit <http://sagemath.org>

## CRM-Fields-PIMS Prize 2008 Awarded To Allan Borodin

The **Centre de recherches mathématiques** (CRM), the **Fields Institute**, and the **Pacific Institute for the Mathematical Sciences** (PIMS) have awarded the 2008 CRM-Fields-PIMS Prize to Allan Borodin (University of Toronto) in recognition of his exceptional achievement.

Professor Borodin is a world leader in the mathematical foundations of computer science. His influence on theoretical computer science has been enormous, and its scope is very broad. Jon Kleinberg, winner of the 2006 Nevanlinna Prize, writes of Professor Borodin, “He is one of the few researchers for whom one can cite examples of impact on nearly every area of theory, and his work is characterized by a profound taste in choice of problems, and deep connections with broader issues in computer science.” Professor Borodin has made fundamental contributions to many areas, including algebraic computations, resource tradeoffs, routing in interconnection networks, parallel algorithms, online algorithms, and adversarial queuing theory.

Professor Borodin received his B.A. in Mathematics from Rutgers University in 1963, his M.S. in Electrical Engineering & Computer Science in 1966 from Stevens Institute of Technology, and his Ph.D. in Computer Science from Cornell University in 1969. He was a systems programmer at Bell Laboratories in New Jersey from 1963-1966, and a Research Fellow at Cornell from 1966-1969. Since 1969 he has taught with the computer science department at the University of Toronto, becoming a full professor in 1977, and chair of the department from 1980-1985. Professor Borodin has been the editor of many journals including the *SIAM Journal of Computing*, *Algorithmica*, the *Journal of Computer Algebra*, the *Journal of Computational Complexity*, and the *Journal of Applicable Algebra in Engineering, Communication and Computing*. He has held positions on, or been active in, dozens of committees and organizations, both inside and outside the University, and has held several visiting professorships internationally.

Borodin was elected a Fellow of the Royal Society of Canada in 1991.

Previous recipients of the prize are H.S.M. (Donald) Coxeter, George A. Elliott, James Arthur, Robert V. Moody, Stephen A. Cook, Israel Michael Sigal, William T. Tutte, John B. Friedlander, John McKay, Edwin Perkins, Donald A. Dawson, David Boyd, Nicole Tomczak-Jaegermann and Joel Feldman.

Established in 1994, the CRM-Fields Prize recognizes exceptional research in the mathematical sciences. In 2005, PIMS became an equal partner in the prize, and the name was changed to the CRM-Fields-PIMS prize. A committee appointed by the three institutes chooses the recipient.

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# CRM-Fields-PIMS Prize 2008

## Allan Borodin

by Stephen Cook (U.Toronto)

Allan Borodin of the University of Toronto has been awarded the 2008 CRM-Fields-PIMS prize. According to the citation, “*Professor Borodin is a world leader in the mathematical foundations of computer science. His influence on theoretical computer science has been enormous, and its scope very broad. Jon Kleinberg, winner of the 2006 Nevanlinna Prize, writes of Dr. Borodin, ‘He is one of the few researchers for whom one can cite examples of impact on nearly every area of theory, and his work is characterized by a profound taste in choice of problems, and deep connections with broader issues in computer science.’ Allan Borodin has made fundamental contributions to many areas, including algebraic computations, resource tradeoffs, routing in interconnection networks, parallel algorithms, online algorithms, and adversarial queuing theory.*”

Dr. Borodin received his B.A. in Mathematics from Rutgers University in 1963, his M.S. in Electrical Engineering & Computer Science in 1966 from Stevens Institute of Technology, and his Ph.D. in Computer Science from Cornell University in 1969. He was a systems programmer at Bell Laboratories in New Jersey from 1963–1966, and a Research Fellow at Cornell from 1966–1969. Since 1969, he has taught with the computer science department at the University of Toronto, becoming a full professor in 1977 and chair of the department from 1980–1985. He has been the editor of many journals including the *SIAM Journal of Computing*, *Algorithmica*, the *Journal of Computer Algebra*, the *Journal of Computational Complexity*, and the *Journal of Applicable Algebra in Engineering, Communication and Computing*. He has held positions on, or been active in, dozens of committees and organizations, both inside and outside the University, and has held several visiting professorships internationally. In 1991, Dr. Borodin was elected a Fellow of the Royal Society of Canada.

The discipline of computer science has been an exceptionally successful blend of engineering and mathematical science with a healthy dose of human factor and aesthetic issues. Dr. Borodin has made significant contributions to many diverse aspects of the discipline, with a major focus on the more mathematical areas. A common theme in his research is that he explores fundamental questions that should have well-understood explanations, but seem to often defy answers to even the most basic forms of these questions. As a result, Dr. Borodin has often been at the forefront of developing new models and problem formulations that have become standard frameworks for studies in computer science.

Perhaps the most basic scientific aspect of computer science is to understand the intrinsic limitations of what can and what cannot be *efficiently* computed in various models of computing with respect to various measures of complexity. This study is the heart of complexity theory. The other side of the complexity theory coin is the design and analysis of algorithms.

Dr. Borodin has been involved in both sides of this coin since his first publication in 1969. In his Cornell Ph.D. thesis, Dr. Borodin studied the time complexity classes introduced by Hartmanis and Stearns and the more abstract complexity measures axiomatized by Blum. He showed that “constructible” bounding functions as used by Hartmanis and Stearns to develop complexity hierarchies are necessary in the sense that for any complexity measure (be it time, space, etc.) there are arbitrarily large gaps (where no new functions are being computed) created by non-constructible bounding functions [Bor72]. Another thesis result (with Constable and Hopcroft) showed that time complexity classes are dense [BCH69].

Dr. Borodin soon became more focused on the complexity of specific functions and, in particular, what we now call “algebraic complexity theory.” The complexity world was basically uncharted territory at the end of the 1960s, although many surprising and widely applicable results (for example, the Fast Fourier Transform and fast integer multiplication) were developed outside the confines of a formal theory. A number of results accelerated the development of complexity theory. One such result was Cook’s formulation of the class NP and the identification of NP complete problems which became and still remains the main source of evidence that many common combinatorial problems cannot be solved efficiently (i.e. within polynomial time). On the other side of the coin, Strassen’s surprising result that matrix multiplication can be computed within  $O(n^{\log_2 7}) \approx O(n^{2.81})$  arithmetic operations showed that our common intuition and beliefs cannot be trusted (the obvious method requires  $n^3$  multiplications). Following Strassen’s dramatic result, Dr. Borodin proved a number of results helping to establish the field of algebraic complexity. Resurrecting an old question raised by Ostrovsky, Dr. Borodin showed that Horner’s rule for evaluating a polynomial is uniquely optimal in being the only method that can achieve the optimal  $2n$  arithmetic operations. Since (even with preconditioning)  $n/2$  multiplications/divisions and  $n$  additions/subtractions are required for one polynomial evaluation for most degree  $n$  polynomials, how many operations are needed to evaluate a degree  $n$  polynomial at (say)  $n$  arbitrary points? When the evaluation points are the powers of a suitable primitive root of unity, the FFT performs these evaluations in  $O(n \log n)$  operations rather than the  $O(n^2)$  operations required if one evaluates at each point individually. By reduction to Strassen’s fast matrix multiplication, Borodin and Munro showed that  $O(n^{1.91})$  operations are sufficient [BM71]. Then Borodin and Moenck showed that  $\Omega(n \log n)$  non scalar multiplications and  $O(n \log^2 n)$  total operations are sufficient [BM74], which remains the best asymptotic bound for total operations (and matched by Strassen’s algebraic geometry based  $\Omega(n \log n)$  lower bound for the number of non scalar multiplications). The Strassen bound uses the Bezout Theorem on

Please check out the PIMS website for news on the 2009 CRM-Fields-PIMS Prize, <http://www.pims.math.ca>





Allan Borodin  
Photo courtesy of the Fields Institute

the degree of an algebraic variety to generalize the obvious fact that an  $n^{\text{th}}$  degree polynomial requires  $\log n$  multiplications (since the degree can at most double following a multiplication).

Using the FFT, two  $n^{\text{th}}$  degree polynomials can be multiplied in  $O(n)$  non scalar multiplications and  $O(n \log n)$  additions. Is there an analogue to the degree bound so as to establish lower bounds on the number of additions to compute polynomials? Borodin and Cook [BC76] show that the number of *real* roots of a polynomial is bounded by the minimal number of additions used to compute the polynomial. The Borodin and Cook lower bounds were improved by Risler using results from real algebraic geometry. Beyond these research contributions, Dr. Borodin and his first Ph.D. student Ian Munro wrote the seminal text book [BM75] in the area of algebraic complexity, which remained the most authoritative source for approximately 20 years.

Another area of interest for Dr. Borodin concerns parallel computation and network routing. How does parallel time complexity relate to the more standard complexity measures of time and space? Following the known results relating sequential time with uniform circuit size, Dr. Borodin showed that the space measure is directly related to uniform circuit depth, a basic measure of parallel complexity. Unlike the situation for classical sequential time studies, there are alternative models of parallel computation, including various parallel RAM models and interconnection network models. In order for an interconnection network to be able to simulate a RAM, the network must be able to simply and efficiently rout simultaneous messages. Oblivious routing schemes are simple in the sense that the path of each message is independent of the routes of other messages. Valiant showed that by obliviously routing to a random intermediate node, any permutation could be routed in time  $O(d)$  on a  $d$ -dimensional hypercube. This is asymptotically optimal since  $d$  is the diameter of the network. Borodin and Hopcroft [BH85] showed that this use of a random intermediate node is necessary in the following strong sense: in any degree  $d$  network with  $n$  nodes, for any deterministic (i.e. non randomized) oblivious routing algorithm, there exists a permutation that will have a bottleneck node forcing the routing to take at least  $\Omega(\sqrt{n}/d^{3/2})$  time. Dr. Borodin was also the co-designer of some surprising parallel algorithms, including (with von zur Gathen and Hopcroft) [BvzGH82] a randomized parallel greedy algorithm

to derive a  $\log^2 n$  parallel time (i.e. depth of arithmetic circuit) algorithm for computing the rank of an  $n \times n$  matrix, and a  $\log \log n$  algorithm for merging two lists on a CREW (Concurrent Read ExclusiveWrite) RAM model. The work on packet routing led to a new area of research. Packet routing can be viewed as a queuing system in which the edges of the network become the processes and one can study the queuing effects in terms the nature of the network and/or the scheduling rules used by the nodes of the network. In this setting, input requests (e.g. oblivious packet paths or requests for packet transmission along any path from source to target) are characterized more by burstiness rather than by any standard probabilistic distribution. Furthermore, the processing time (transmission of a single or a bounded number of packets along an edge) is usually considered to have a well-defined time. Borodin, Kleinberg, Raghavan, Sudan and Williamson [BKR<sup>+</sup>01] modeled this burstiness by an adversarial model and developed an area named “adversarial queuing theory.” There are some natural queuing limitations on the stability (i.e. bounded queue sizes and time to complete a transmission) with the main limitation (say in oblivious routing) being that the rate of requests for an edge cannot exceed the processing rate of the edge. Borodin et al. began the study as to which networks are always stable at a given rate (independent of the scheduling rule) and which scheduling rules are always stable (independent of the network). Adversarial studies of packet routing and other queueing systems has led to a number of surprising results (e.g. the instability of FIFO at any rate for certain networks as shown by Bhattacharjee and Goel).

While complexity theory has been very successful in many aspects (e.g. understanding the relation between complexity measures, establishing complexity based cryptography, utilizing hardness to develop pseudo random generators, the development of new notions of “proofs” including interactive proofs and probabilistically checkable proofs), the major limitation of the field thus far is in the inability to prove complexity impossibility results for “explicitly defined natural problems” (for example, NP search and optimization problems). More specifically, non-linear time bounds (on a sufficiently general model of computation) or space bounds greater than  $\log n$  still elude us. Perhaps then the simplest barrier to break is to exhibit a problem which cannot be simultaneously computed in small time and space. Dr. Borodin led a group of coauthors [BFK<sup>+</sup>81] to prove the first time-space tradeoff result for comparison based sorting in what can be said to be the most general model for such a result. They considered comparison branching programs which are DAGS where nodes are labelled by comparisons “ $a_i \leq a_j$ ?” between elements from a given “read-only” input set of  $n$  elements. In this model, edges are labelled by sequences of input elements that are being output if this edge is traversed. The complete sequence of outputs along any path defines the output of the program. In this non-uniform model (like circuits, a different program is allowed for each  $n$ ), time is the length of the longest path (or expected path length if one were considering average case complexity), and space is the logarithm of the number of nodes in the DAG (i.e. the information theoretic lower bound on the memory being used). In contrast to list merging, which can be computed simultaneously in linear time and  $O(\log n)$  space, Borodin et al. show that the time space product  $T \cdot S = \Omega(n^2)$ ; that is, any small space method must require significantly more time than the optimal  $n \log n$  bound achievable by methods such as merge-sort. (For all space bounds  $S(n)$  between  $\log n$  and  $n$ , a corresponding upper bound can be obtained.)

This paper [BFK<sup>+</sup>81] was seminal and started a long and continuing research effort to derive time space bounds for natural problems in appropriate models. The sorting tradeoff was soon followed by a similar



comparison branching program tradeoff for a decision problem, namely the element distinctness problem. The initial element distinctness tradeoff was by Borodin et al. [BFadH<sup>+</sup>87], and it was then improved by Yao. These comparison branching programs (where the algorithm does not have access to the encoding of the input elements) leaves open the possibility that the corresponding Boolean problems (e.g. say encoding integer inputs in binary) can be computed using simultaneously small time and space. This consideration led Borodin and Cook [BC82] to introduce the  $R$ -way branching program model, where now inputs are considered to be inputs in some range  $[1, R]$ , and branching program nodes are of the form “ $a_i = ?$ ”, with up to  $R$  branches corresponding to each of the possible values of  $a_i$ . Time and space are defined as before. Borodin and Cook showed that sorting  $n$  numbers in the range  $[1, n^2]$  required  $T \cdot S = \Omega(n^2)$ , proving a very strong tradeoff result, since the total binary encoding length of the input is only  $O(n \log n)$  bits. This represents the first negative result for an explicit (polynomial time computable) Boolean problem in a completely general model, albeit not a decision problem. It took approximately 20 more years to establish negative results (of a much weaker form) for a Boolean decision problem.

In the mid 1980s, Dr. Borodin began working on the topic of online approximation algorithms which became known as competitive analysis, whereby the performance of an online algorithm (making decisions for each input as it arrives) is compared to the performance of an optimal solution with complete knowledge of the entire input. There had been a number of earlier important results concerning online algorithms for specific problems that need not necessarily be considered as online problems (for example, Graham’s study of the makespan problem, Kierstead and Trotter’s online interval colouring, and Yao’s study of online bin packing). Sleator and Tarjan proposed competitive analysis (in contrast to distributional studies) for problems that were inherently online such as paging and list accessing. Borodin, Linial and Saks [BLS92] then proposed an abstract online problem framework called metrical task systems (MTS) which was soon followed by the  $k$ -server model of Manasse, McGeouch and Sleator. The introduction of competitive analysis for online problems and these abstract problem formulations spawned a wealth of research activity that has had an impact well beyond online problems. For example [BLS92] provides an optimal  $2n - 1$  competitive ratio bound for deterministic algorithms for any  $n$ -state MTS. It also introduced randomized algorithms in this context showing that the uniform metric system had a  $2H_n \approx 2 \ln n$  randomized competitive ratio. This led the way to a randomized paging algorithm by Fiat et al. and, moreover, led to interest in trying to derive randomized algorithms for general MTS and  $k$ -server problems. In this context Bartal introduced Hierarchically Separated Tree spaces (HSTs) for which  $O(\log n)$  randomized algorithms exist and furthermore arbitrary metric spaces can be efficiently embedded into such HSTs. The use of HSTs has now become a standard tool in combinatorial approximation.

Beyond the seminal MST work, Dr. Borodin was influential in a number of central results concerning online competitive analysis. Borodin et al. [BIRS95] introduced a variant of competitive analysis so as to model the locality of reference exhibited by (for example) paging requests. Another landmark paper introduces “request–answer games,” which provide a framework for defining most known online problems. In this very abstract setting (which, for example, includes the MTS and  $k$ -server settings), Borodin and coauthors [BDBK<sup>+</sup>94] relate the power of different adversarial models for randomized online algorithms; namely, they identify the more standard oblivious adversary (as used in offline computation) where

the adversary generates the input request sequence without knowledge of the algorithm’s coin tosses, and adaptive adversaries where the adversary adaptively creates the input sequence by observing the coin tosses and actions of the online algorithm. For adaptive adversaries, the adversary (acting also as the “optimal benchmark”) can either play the game online or play the game in hindsight as an offline player. A number of randomized algorithms were being studied relative to (not so precisely defined) adaptive adversaries. Ben-David et al. show that algorithms competing against online adaptive adversaries can be simulated by algorithms competing against offline adaptive adversaries which in turn can be simulated by deterministic algorithms thereby showing that randomization can only yield significantly improved competitive ratios when formulated as algorithms competing against oblivious adversaries.

Finally one of Dr. Borodin’s most influential contributions to online analysis is his text [BEY98] with former student Ran El-Yaniv. The text (published in 1998) remains the authoritative reference for this area, although many significant results have followed its publication, including a number of results addressing questions raised in the book.

Dr. Borodin has made significant contributions to a number of other aspects of algorithm analysis. One paper with Ostrovsky and Rabani [BOR03] provides the first memory–search time results for problems (e.g. nearest neighbour and partial match search) in high dimensional spaces, proving that for deterministic algorithms some form of exponential “curse of dimensionality” must exist for a widely studied geometric search model.

Dr. Borodin’s most recent research area has been an area he has essentially been creating, namely the attempt to study the power and limitations of “simple algorithms”, especially (to date) for search and optimization problems. While we equate efficient algorithms with time and/or memory efficiency, there are other important aspects to algorithm design. It is a rather remarkable fact that for over 70 years we have a well-accepted formalization (i.e. the Church–Turing thesis) for the intuitive concept of “computable function” and the associated concept of an algorithm. And if we stay within classical computing models (in contrast to say quantum computers) we have a reasonably well-accepted definition of “efficiently computable.” But we often want simple understandable algorithms, at least as starting points or benchmarks for developing more sophisticated, complex algorithms. That is, we tend to use a small set of basic algorithmic paradigms as a “toolbox” for an initial (and sometimes the best known or even optimal) method for solving large classes of problems in many settings. These basic paradigms include greedy algorithms, divide and conquer, dynamic programming, local search, primal dual algorithms and IP/LP rounding. Surprisingly, although we intuitively understand what these concepts mean, rarely do we attempt any precise formulation, and a precise formulation is necessary if one is to gain any insight into the ultimate power and limitations of these methods.

The long-standing and significant study and use of greedy algorithms provides a great example of an algorithmic paradigm that seems so natural and obvious that no definition seems necessary. It is hard to think of a computational area where some concept of greediness does not appear. The elegant results of Edmonds, Korte, Lovasz connecting matroids and greedoids with the optimality of “the” natural greedy algorithm for certain set systems was the starting point for a number of insightful results concerning optimal greedy algorithms. But greedy algorithms are mainly used as a heuristic or to obtain approximation results. Borodin, Nielson and Rackoff [BNR03] introduce the priority algorithm framework as a model for “greedy-like” optimization algorithms in almost any setting. We can

think of this framework as an offline extension of online algorithms. An input to a problem is a set of items (for example, jobs in a scheduling problem, vertices in a graph problem, propositional variables in a SAT problem) and a priority algorithm considers and makes decisions about items one by one but now in an order determined (in advance or adaptively) by the algorithm rather than the order given by (adversarial) nature. Of course, the trick here is formulate what orderings a “reasonable” algorithm can use. For example, it would make no sense to allow the algorithm to compute an optimal solution and a corresponding optimal order that allows the algorithm to produce the optimal solution. One approach would be to resort to complexity considerations and say that each item is chosen within some acceptable time. But that would bring us back to our current inability to prove limitations based on time complexity. Instead the priority framework relies on a simple to state concept of a local ordering. In fact, the allowable orderings are (at each iteration in the case of adaptive priority algorithms) those satisfying Arrow’s IIA (independence of irrelevant attributes) axiom. Whereas in social choice theory this axiom is controversial, for greedy-like algorithms the concept allows great generality while still being amenable to analysis. And what does this have to do with greediness? In the priority framework it is not the ordering decisions that are greedy but rather (for greedy priority) it is the decisions being made for each input item that can be construed as greedy (say in the sense of “living for today”) with respect to the given objective function. There are a number of results showing the limitations of such priority algorithms in different domains, starting with the initial scheduling results of Borodin, Nielson and Rackoff.

The priority framework is also the starting point for more powerful paradigms, such as some simple forms of primal dual algorithms using a reverse delete step, simple dynamic programming and backtracking. For example, the work of Dr. Borodin and coauthors [ABBO<sup>+</sup>05] shows why DPLL style backtracking algorithms cannot solve 3SAT search and has limits to approximating Max2Sat but can solve 2SAT. They also show that the form of dynamic programming used for interval scheduling and knapsack algorithms have limitations. In particular, optimal dynamic programming algorithms for weighted interval scheduling on  $m$  machines must suffer a curse of dimensionality with respect  $m$ .

This recent algorithmic design work reflects the style of an extraordinarily productive and creative career.

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# Ten Years of Mathematical Excellence: *PIMS' First Decade*

In 2006-2007, PIMS celebrated its 10<sup>th</sup> Anniversary. The PIMS universities hosted a series of distinguished speakers, bringing celebrated international mathematical talent to the PIMS community. A complete list of speakers of all PIMS' Anniversary Distinguished Speakers is available on page \*\*, and at <http://www.pims.math.ca>.

The University of Victoria hosted the Symposium on Kinetic Equations and Methods in April, 2007, as its celebration of the PIMS 10th Anniversary. For more on this conference, please see the scientific report on page \*\*.

The culmination of the year's celebrations took place on September 27, 2007, at the University of British Columbia. The event began with a scientific symposium. Daniel Pauly (UBC) presented *Fisheries and Global Warming: Impacts on Marine Ecosystems and Food Security*. Peter Sarnak (Institute for Advanced Study and Princeton University), spoke on *Equidistribution and Primes*. An expanded paper based on Dr. Sarnak's lecture is found on page \*\* of this issue.

The banquet, in the First Nations Longhouse at the University of British Columbia, was attended by over 150 academics, dignitaries and friends of PIMS. Brian Russell (CGG Veritas), chairman of the PIMS board, spoke to the wonderful milestones PIMS has achieved over its first decade, including:

- The Alberta government has more than doubled its support for PIMS, with a new three-year grant for 2007-2010.
- The British Columbia government gave \$130,000 to PIMS to help teachers increase math participation of students in First Nations schools.
- PIMS has been renewed by NSERC for a five-year period, 2008-2013, and its grant increased by 10 percent. Dr. Russell expressed PIMS' gratitude to NSERC for their continuing support.
- PIMS has become an Unité Mixte Internationale of the French CNRS. This is a symbol of international recognition.
- The University of Regina is becoming a full member of PIMS.
- PIMS, CINVESTAV and the Mexican Mathematical Society, planned a conference in Monterrey on the mathematics of oil exploration, with the help

and participation of CGG-Veritas.

- Lastly, Dr. Russell spoke of the increasingly strong ties between industry and PIMS. One example familiar to the Chairman of the Board, who is a geophysicist, is the inversion initiative spearheaded by Gary Margrave (U.Calgary), Gunther Uhlmann (U.Washington) and Felix Hermann (UBC), which has resulted in several high-level inversion seminars and conferences, and lead to algorithms used by industry.

Other speakers at the banquet praised PIMS' activities over the decade, including Arvind Gupta, MITACS Director; Nassif Ghoussoub, BIRS Director and founder of PIMS; Hugh Morris, past chairman of the PIMS Board; Andy Greenshaw, Associate Vice-President Research, U.Alberta; Ron Irving, Dean of Arts and Science at U.Washington; and Haig Farris, member of the PIMS Board.

Ivar Ekeland, PIMS Director, ended the evening by thanking all for attending the event, and wished PIMS a successful second decade.



The PIMS 10<sup>th</sup> Anniversary Event in full swing  
Photograph by Tom Brauer

## Peter Lax on Collaborations in Mathematics

*Peter Lax is Professor Emeritus of Mathematics and former Director, Courant Institute, New York University. He has made significant contributions to both mathematics and computing. Dr. Lax was awarded the Abel Prize "for his ground-breaking contributions to the theory and application of partial differential equations and to the computation of their solutions," and has received numerous other honours including the National Medal of Science, the Lester R. Ford Award, the Chauvenet Prize, the Semmelweis Medal, the Wiener Prize and the Wolf Prize.*

*He presented a lecture for the PIMS 10<sup>th</sup> Anniversary Lecture Series, at the University of Vancouver; and spoke with PIMS on how his mathematical path has been a collaborative one.*

### Who were the people who most influenced you in your mathematical training?

I have been extremely fortunate in receiving help along every step of my education. There is a tradition in Hungary to recognize early and foster interest and talent in mathematics. My first mentor was my uncle, Albert Kornfeld, himself a winner of the Eotvos competition in 1916. I then was tutored for three years by Rose Peter, best known as the author of the outstanding popular math book "Playing with Infinity." When we came to America in December of 1941, she and Denes Koenig, wrote to von Neumann to look after me. The parents of Erdos sent a letter to Paul, then at Princeton, to talk mathematics with me, which he very kindly did.

Gabor Szego, a good friend of my parents, suggested that I be placed with Richard Courant, who was very good at nurturing young people; I never received better advice. After finishing high school (Stuyvesant) in January 1943, I entered NYU; I was ready for graduate training in mathematics. My college career was interrupted by the draft into the US Army when I turned 18. After my basic training I was sent to study engineering to Texas A&M. My relatively advanced training in mathematics was noted, and Dr Clyde Klipple, a former student of R.L. Moore, very kindly trained me in real variables by the Moore method.

From Texas A&M, I was transferred in the late spring of 1945 to Los Alamos to work on the Manhattan Project; it was like living science fiction. I stayed there a year, and after my discharge in 1946, I resumed my educa-



tion, graduate in mathematics, undergraduate in everything else. I received my bachelor degree in 1947, and my Ph.D. in 1949, under the direction of Friedrichs. My classmates in graduate school were Louis Nirenberg, Cathleen Morawetz, Joe Keller, Harold Grad, Avron Douglis and Martin Kruskal.

In the fall of 1949 I took a staff position at Los Alamos, then in the throes of building a hydrogen bomb. I returned to NYU as a research assistant professor a year later, but I

spent most of my Summers at Los Alamos; thus did I get involved, under the influence of von Neumann, in computational fluid dynamics, and numerical analysis.

This brings me to the end of my formative years; I will mention my debt to Szego and Polya during many Summer visits to Stanford, where I had the good fortune to meet and start collaborating with Ralph Phillips. At Los Alamos I was drawn to the offbeat mathematical personality of Stan Ulam. Contact with the international mathematical community, with Lars Garding, Lars Hormander, Ludvig Faddeev, Israel Gohberg, Gelfand, Leray, Lions, Brezis and others have shaped my mathematical outlook.

My deepest gratitude goes to the my teachers, colleagues, and students at the Courant Institute.

# PIMS 10<sup>th</sup> Anniversary Speakers



A complete list of the PIMS 10<sup>th</sup> Anniversary speakers, with titles to their lectures. For abstracts and multimedia, please see [http://www.pims.math.ca/PIMS\\_10th\\_Anniversary\\_Activities/](http://www.pims.math.ca/PIMS_10th_Anniversary_Activities/)

## Simon Fraser University

- Alexander Razborov (IAS) *Feasible Proofs and Computations*  
Jesper Lützen (U.Copenhagen) *Mechanistic Images in Geometric Form: Heinrich Hertz's Principles of Mechanics*  
George C. Papanicolaou (Stanford) *Imaging in random media*  
David Brillinger (UC Berkeley) *A unified approach to modelling trajectories*  
Craig Evans (UC Berkeley) *A Nonlinear PDE Model for Lakes and Rivers*  
Efim Zelmanov (UC San Diego) *On profinite groups*  
Herbert Wilf (U.Pennsylvania) *Mathematics: An Experimental Science*  
John Mason (Open University) *Mathematical Pedagogy and Pedagogical Mathematics*  
Paul Seymour (Princeton) *Structure Theorems in Graph Theory*

## University of Alberta

- Mark Chaplain (U.Dundee) *Mathematical Modelling of Host-Parasitoid Systems: Spatio-Temporal Dynamics and Mathematical Modelling of Cancer Growth and Progression: Angiogenesis and Invasion*  
Benoit Perthame (École Normale Supérieure) *Adaptive evolution and concentrations in parabolic PDEs*  
Shuji Saito (U.Tokyo) *Chow group of 0-cycles on a surface over a p-adic field with infinite torsion subgroup*

## University of British Columbia

- Andrei Okounkov (Princeton) *Frozen boundaries and log-fronts*  
Helmut Hofer (Courant Institute) *New geometric and functional analytic ideas arising from problems in symplectic geometry*  
Raman Parimala (Emory U./Tata Institute) *Sums of Squares and Pfister forms*  
Mark Lewis (U.Alberta) *Plagued by numbers: the mathematics of disease*  
Jerry Sacks (NISS) *The Reality of Computer Models: Statistics and Virtual Science*  
Gary Leal (UCSB) *Computational Studies of the Motion of a Nematic LCP in a Simple Shear Device*  
James Berger (UC Berkeley) *Issues with Bayesian Analysis of Inverse Problems*  
Nancy Reid (U.Toronto) *The interface between Bayesian and frequentist statistics*  
Michael Stein (U.Chicago) *Statistical Models for Global Processes*  
Peter Lax (Courant Institute) *Asymptotic behavior at infinity of solutions of elliptic equations*  
Gunnar Carlsson (Stanford) *Algebraic Topology and Geometric Pattern Recognition*  
Tai-Ping Liu (Stanford) *Prandtl Conjecture and von Neumann Paradox for Shock Reflections*  
Darrell Duffie (Stanford) *Frailty Correlated Default*

## University of Calgary

- John Taylor (U.Montreal) *The integral geometry of random sets*  
Robert J. Adler (Technion) *The brain, the universe, and random processes on manifolds*  
Richard Howitt (UC Davis) *A Computational Economics Approach to Policy Models: Applications to Natural Resources*  
Karlheinz Groechenig (U.Vienna) *Time-Frequency Analysis: From Wireless Communications to Abstract Harmonic Analysis*  
Mikael Passare (U.Stockholm) *Amoebas, Coamoebas, and Tropical Geometry*  
Bernhard Schmidt (Nanyang Technological University) *Values and Ideals in Combinatorial Problems and Gauss Sums: Finding the Root of Unity*

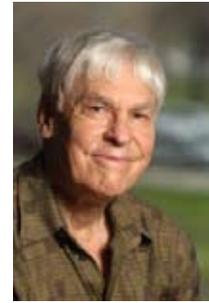
## University of Washington

- Gregory Lawler (U.Chicago) *Conformal Invariance and Two-dimensional Statistical Physics*  
Peter Bickel (UC Berkeley) *Regularized covariance matrix estimation*  
Bin Yu (UC Berkeley) *Feature Selection through Lasso: model selection consistency and the B-Lasso algorithm*  
Stephen Smale (TTI-C) *Topology, data and vision and Emergence and Flocking*  
Klaus Schmidt (U.Vienna) *On Some of the Differences Between  $Z$  and  $Z^2$  in Dynamics*  
Jim Zidek (UBC) *Statistical modeling in setting air quality standards*  
Peter Lax (Courant Institute) *A Phragmen-Lindelof and Saint Venant principle in harmonic analysis*  
Elliott Lieb (Princeton) *The Dilute, Cold Bose Gas: A truly quantum-mechanical many-body problem and Quantum Mechanics, the Stability of Matter, and Quantum Electrodynamics*  
Carlos Kenig (U.Chicago) *Recent developments on the well-posedness of dispersive equations*  
Peter Winkler (Dartmouth College) *Scheduling, Percolation, and the Worm Order*  
Richard Schoen (Stanford) *The sharp isoperimetric inequality on minimal submanifolds*  
Chris Jones (U.North Carolina) *Going with Flow and Updating Ocean Models*  
Frances Kirwan (U. Oxford) *Classification problems in algebraic geometry and Non-reductive group actions and symplectic implosion*  
Leo Kadanoff (U.Chicago) *The Good, the Bad, and the Awful: Scientific Simulation and Prediction*  
Shrawan Kumar (U.North Carolina) *Eigenvalue Problem and a New Product in Cohomology of Flag Varieties*  
Leo Kadanoff (U.Chicago) *Making a Splash, Breaking a Neck: The development of complexity in physical systems*  
Kari Astala (U.Helsinki) *Mappings of finite distortion: analysis in the extreme*



# An Interview With Steve Smale

*Steve Smale, mathematician with the Toyota Technological Institute at Chicago, sat down with PIMS while he was at the University of British Columbia for the First International Congress of IPIA, Conference on Applied Inverse Problems 2007: Theoretical and Computational Aspects*



Steve Smale (Toyota Technological Institute)

**PIMS:** Can you tell us a bit about your background and the areas of math that you are currently interested in?

**Smale:** What I'm interested in now is mainly connecting with learning theory in mathematics and science. I'm working a lot on that, and an area called flocking, a model on how birds flock together. It has to do with vision, the mathematics of vision.

**PIMS:** How long have you been working in these areas? Are these recent areas of interest?

**Smale:** I've been focusing on learning theory for about seven or eight years. With vision, it has been directly for one year; it's related to learning theory. As for flocking, two years.

**PIMS:** What brought you to these areas?

**Smale:** They are a kind of mathematics which are motivated by the world in some sense. Trying to find a mathematical foundation for natural phenomenon, develop those mathematical foundations. That's the kind of direction it is. It's not pure math and it's not applied math. I use a model and an inspiration people like Newton and Neiman, who have the ideas for the mathematical foundations of physics. It's the foundations where one can see much of the universality; an apple falling on Newton is the same phenomenon as the motion of the planets. It's the universality and simplicity at the same time, so those are Neiman's foundations of quantum mechanics. Another model for me is where we introduce Hilbert spaces and operators.

That's the kind of things that I try to do. It isn't what I'd call applied math, exactly, nor is it pure math, same with Neiman's work on quantum mechanics. That's the place I put myself. I don't put myself with those two people, but they are the inspiration for my point of view.

**PIMS:** The field of learning theory, in which you are currently working, how long has it been around?

**Smale:** Some versions have been around for ages. But it's had some new developments, and now it's called learning theory. Before that it was called sampling theory. A lot of different things have gone into this and it has segued. It was sampling theory way back; a lot of the ideas that I use came from Gauss.

**PIMS:** You started off at the University of California Berkeley, and now you're in Chicago at the Toyota Technological Institute at Chicago. How long have you been there?

**Smale:** About four and a half years, with the University of Chicago. I have position there. As well, I'm a professor emeritus at Berkeley.

**PIMS:** What is your talk at the *Conference on Applied Inverse Problems* going to be about?

**Smale:** Learning Theory.

**PIMS:** You also presented two talks at the University of Washington in January, 2007, at the PIMS 10<sup>th</sup> Anniversary lecture series.

**Smale:** Yes, on *Topology, data and vision* (Abstract: Large data sets usually have some geometric core. We give some results toward understanding that core) and *Emergence and Flocking*. (Abstract: We give a mathematical model with ordinary differential equations and the graph laplacian towards the problem: what decentralized mechanisms give rise to common features as language? A critical exponent plays a role in this deterministic setting).

**PIMS:** You have a large collection of minerals. Tell us about that.

**Smale:** It's one of the best collections in the world, but not the largest, because we focused more quality than quantity.

**PIMS:** What started you in collecting, more than 30 years ago?

**Smale:** It's hard to know all the causes, but certainly the beauty of the minerals. We have a new book, entitled *The Smale Collection: Beauty in Natural Crystals*, a coffee table book with one hundred full-page photographs. The collection itself is at Berkeley.

**PIMS:** Can you sum up learning theory in a few words?

**Smale:** The one thing that I did was in my first paper on learning theory, called "The Mathematical Foundations of Learning", in which we tried to get some sort of systematic simple foundations for a subject that had been developed by many, many scientists outside of mathematics, doing just what the word *learning* describes — people in computer science, psychology, engineering, statistics. There has been a lot of work over the last 10-20 years, which had to do with how machines learn, how people learn, how animals learn, and what we were trying to do (and are still doing) with the mathematical foundations was trying to find these universal ideas of learning, which apply not just to people but to other animals, to machines, trying to find out what is universal about learning. The idea is learning examples; learning by examples suggests sampling theory and also it related to inverse problems, and I suppose that's why I'm here [at the AIP conference], because of that sampling aspect is learning is inductive learning, learning from a lot of examples, it's a lot of traditions in inverse problems and sampling theory before that. In the combined way that I see it, learning theory brings together probability theory and approximation theory. The most characterizing property of learning theory was the great synthesis of probability and approximation theory in a natural way.

# Equidistribution and Primes

by Peter Sarnak

(Princeton University & Institute for Advanced Study)

## Expanded Version of the Pacific Institute for the Mathematical Sciences 10<sup>th</sup> Anniversary Lecture, September 27, 2007

### Abstract

We begin by reviewing various classical problems concerning the existence of primes or numbers with few prime factors as well as some of the key developments towards resolving these long-standing questions. Then we put the theory in a natural and general geometric context of actions on affine  $n$ -space and indicate what can be established there. The methods used to develop a combinatorial sieve in this context involve automorphic forms, expander graphs and unexpectedly arithmetic combinatorics. Applications to classical problems such as the divisibility of the areas of Pythagorean triangles and of the curvatures of the circles in an integral Apollonian packing, are given.

(1) I have chosen to talk on this topic because I believe it has a wide appeal and also there have been some interesting developments in recent years on some of these classical problems. The questions that we discuss are generalizations of the twin prime conjecture; that there are infinitely many primes  $p$  such that  $p+2$  is also a prime. I am not sure who first asked this question but it is ancient and it is a question that occurs to anyone who looks, even superficially, at a list of the first few primes. Like Fermat's Last Theorem there appears to be nothing fundamental about this problem. We ask it simply out of curiosity. On the other hand the techniques, theories and generalizations that have been developed in order to understand such problems are perhaps more fundamental.

(2) Dirichlet's Theorem: In many ways this theorem is still the center piece of the subject. Like many landmark papers in mathematics, Dirichlet's paper proving the theorem below, initiated a number of fields: abelian groups and their characters,  $L$ -functions, class number formulae... . The theorem asserts that an arithmetic progression  $c, c+q, c+2q, c+3q, \dots$  contains infinitely primes if and only if there is no obvious congruence obstruction. An obvious such obstruction would be say that  $c$  and  $q$  are both even or more generally that the greatest common divisor  $(c, q)$  of  $c$  and  $q$  is bigger than 1. Stated somewhat differently, let  $L \neq 0$  be a subgroup of  $\mathbb{Z}$ , so  $L = q\mathbb{Z}$  for some  $q \geq 1$ , and let  $\mathcal{O} = c + L$  be the corresponding orbit of  $c$  under  $L$ , then  $\mathcal{O}$  contains infinitely many primes iff  $(c, q) = 1$  (strictly speaking this statement is slightly weaker since Dirichlet considers one-sided progressions and here and elsewhere we allow negative numbers and call  $-p$  a prime if  $p$  is a positive prime).

(3) Initial Generalizations: There are at least two well-known generalizations of Dirichlet's Theorem that have been investigated. The first is the generalization of his  $L$ -functions to ones associated with general automorphic forms on linear groups. This topic is one of the central themes of modern number theory but other than pointing out that these are used indirectly in proving some of the results mentioned below, I will not discuss them in this lecture. The second generalization is to consider other polynomials besides linear ones. Let  $f \in \mathbb{Z}[x]$  be a polynomial with integer coefficients and let  $\mathcal{O} = c + L$  as above. Does  $f(\mathcal{O})$  contain infinitely many primes? For example if  $\mathcal{O} = \mathbb{Z}$ ; is  $f(x) = x^2 + 1$  a prime number for infinitely many  $x$  (a question going back at least to Euler). Is  $f(x) = x(x+2)$  a product of two primes infinitely often, this is the twin prime question. Neither of these questions have been answered and the answer to both is surely, yes.

We will mention later what progress has been made towards them. In his interesting and provocative article "Logical Dreams" [Sh], Shelah puts forth the dream, that this question of Euler "cannot be decided". This is rather far fetched but for the more general questions about primes and saturation on very sparse orbits associated with tori that are discussed below, such a possibility should be taken seriously. We turn first in the next paragraph to several variables, that being the setting in which some problems of this type have been resolved.

(4) Two Variables: Let  $\mathcal{O} = \mathbb{Z}^2$  and let  $f$  be a nonconstant polynomial in  $\mathbb{Z}[x_1, x_2]$ . If  $f$  is irreducible in  $\mathbb{Q}[x_1, x_2]$  and the greatest common divisor of the numbers  $f(x)$  with  $x \in \mathcal{O}$  is 1, then it is conjectured that  $f$  takes on infinitely many prime values. In this higher dimensional setting we have found it more intrinsic and natural from many points of view to ask for more. That is the set of  $x \in \mathcal{O}$  at which  $f(x)$  is prime should not only produce an infinite set of primes for the values  $f(x)$  but these (infinitely) many points should not satisfy any nontrivial algebraic relation. In the language of algebraic geometry, these points should be Zariski dense in the affine plane  $A^2$ . The Zariski topology on affine  $n$ -space  $A^n$  is gotten by declaring the closed sets to be the zero sets (over  $\mathbb{C}$ ) of a system of polynomial equations. Thus a subset  $S$  of  $A^n$  is Zariski dense in  $A^n$  iff  $S$  is not contained in the zero set of a nontrivial polynomial  $g(x_1, \dots, x_n)$ . In  $A^1$  a set is the zero set of a nontrivial polynomial iff the set is finite. So the Zariski dense subsets of  $A^1$  are simply the infinite sets. We denote the operation of taking the Zariski closure of a set in  $A^n$  by  $Zcl$ .

All the approaches to the conjecture that we are discussing involve giving lower bounds for the number of points in finite subsets of  $\mathcal{O}$  at which  $f(x)$  is prime. Usually one defines these sets by ordering by size of the numbers (so a big box in  $A^2$ ) but in some variations of these problems that I discuss later quite different orderings are employed. A measure of the quality of the process is whether in the end the lower bound is strong enough to ensure the Zariski density of the points produced. As far as the conjecture that under the assumptions on  $f$  at the beginning of (4), the set of  $x \in \mathcal{O}$  at which  $f(x)$  is prime, is Zariski dense in  $A^2$ , the following is known:

- (i) For  $f$  linear it follows from Dirichlet's theorem.
- (ii) For  $f$  of degree two and  $f$  non-degenerate (in the sense of not reducing to a polynomial in one variable) it follows from Iwaniec [Iw]. His method uses the combinatorial sieve which we will discuss a bit further on, as well as the Bombieri-A. Vinogradov theorem



which is a sharp quantitative version of Dirichlet's theorem (when counting primes  $p$  of size at most  $x$  and which are congruent to varying  $c$  modulo  $q$ , with  $q$  as large as  $x^{1/2}$ ).

(iii) A striking breakthrough was made by Friedlander and Iwaniec [F-I]. It follows from their main result that the conjecture is true for  $f(x_1, x_2) = x_1^2 + x_2^2$ . They exploit the structure of this form in that it can be approached by examining primes  $\alpha = a + b\sqrt{-1}$  in  $\mathbb{Z}[\sqrt{-1}]$  with  $b = z^2$ . This was followed by work of Heath-Brown and the results in [H-M] imply that the conjecture is true for any homogeneous binary cubic form. They exploit a similar structure, in that such an  $f(x_1, x_2)$  is of the form  $N(x_1, x_2, 0)$  where  $N(x_1, x_2, x_3)$  is the norm form of cubic extension of  $\mathbb{Q}$ , so that the problem is to produce prime ideals in the latter with one coordinate set to 0.

(iv) If  $f(x_1, x_2)$  is reducible then we seek a Zariski dense set of points  $x \in \mathbb{Z}^2$  at which  $f(x)$  has as few as possible prime factors. For polynomials  $f$  of the special form  $f(x) = f_1(x)f_2(x) \cdots f_t(x)$ , with  $f_j(x) = x_1 + g_j(x_2)$  where  $g_j \in \mathbb{Z}[x]$  and  $g_j(0) = 0$ , it follows from the results in the recent paper of Tao and Ziegler [T-Z] that the set of  $x \in \mathbb{Z}^2$  at which  $f(x)$  is a product of  $t$  primes, is Zariski dense in  $A^2$ . Equivalently the set of  $x$  at which  $f_1(x), \dots, f_t(x)$  are simultaneously prime, is dense. This impressive result is based on the breakthrough in Green and Tao [G-T1] in particular their transference principle, which is a tool for replacing sets of positive density in the usual setting of Szemerédi type theorems with a set of positive density in the primes. The corresponding positive density theorem is that of Bergelson and Leibman [B-L]. Note that for these  $f_j$ 's there is no local obstruction to  $x_1 + g_j(x_2)$  being simultaneously prime since for a given  $q \geq 1$  we can choose  $x_1 \equiv 1(q)$  and  $x_2 \equiv 0(q)$  ( $g_j(0) = 0$ ). Apparently this is a feature of these positive density Szemerédi type theorems in that they don't allow for congruence obstructions.\* The above theorem with  $g_j(x_2) = (j-1)x_2$ ,  $j=1, \dots, t$  recovers the Green-Tao theorem, that the primes contain arbitrary long arithmetic progressions. From our point of view in paragraph (8) the amusing difference between the "existence of primes in an arithmetic progression" and that of an "arithmetic progression in the primes", will be minimized as they both fall under the same umbrella.

(5) **Hardy-Littlewood  $n$ -tuple Conjecture:** This is concerned with  $\mathbb{Z}^n$  and subgroups  $L$  of  $\mathbb{Z}^n$  acting by translations. If  $L$  is such a group denote by  $r(L)$  its rank. We assume  $L \neq 0$  so that  $1 \leq r \leq n$  and also that for each  $j$  the coordinate function  $x_j$  restricted to  $L$  is not identically zero. For  $c \in \mathbb{Z}^n$  and  $\mathcal{O} = c + L$  the conjecture is about finding points in  $\mathcal{O}$  all of whose coordinates are simultaneously prime. We state it as the following local to global conjecture:

**HLC:** If  $\mathcal{O} = c + L$  as above then the set of  $x = (x_1, \dots, x_n) \in \mathcal{O}$  for which the  $x_j$ 's are simultaneously prime, is Zariski dense in  $Zcl(\mathcal{O})$  iff for each  $q \geq 1$  there is an  $x \in \mathcal{O}$  such that  $x_1 x_2 \cdots x_n \in (\mathbb{Z}/q\mathbb{Z})^*$ .

Note that the condition on  $q$ , which is obviously necessary for the Zariski density, involves only finitely many  $q$  (for each given  $\mathcal{O}$ ). Also to be more accurate, the conjecture in [H-L] concerns only the case of  $r(L) = 1$  (which in fact implies the general case). In this case  $Zcl(\mathcal{O})$  is a line and the conjecture asserts that there are infinitely many points in  $x \in \mathcal{O}$  for which the  $n$ -tuples  $(x_1, x_2, \dots, x_n)$  are all prime iff there is no local

obstruction. We observe that for any  $r$  the  $Zcl(\mathcal{O})$  is simply a translate of a linear subspace.

The main breakthrough on the HLC as stated above is due to I. Vinogradov (1937) in his proof of his celebrated "ternary Goldbach theorem", that every sufficiently large positive odd number is a sum of 3 positive prime numbers. His approach was based on Hardy and Littlewood's circle method, a novel sieve and the technique of bilinear estimates, see Vaughan [Va]. It can be used to prove HLC for a non-degenerate  $L$  in  $\mathbb{Z}^3$  of rank at least 2. Special cases of HLC in higher dimensions are established by Balog in [Ba] and recently Green and Tao [G-T2] made a striking advance. Their result implies HLC for  $L \leq \mathbb{Z}^4$  and  $r(L) \geq 2$  and  $L$  non-degenerate in a suitable sense. Their approach combines Vinogradov's methods with their transference principle. It makes crucial use of Gowers' techniques from his proof of Szemerédi's theorem, and it has close analogies with the ergodic theoretic proofs of Szemerédi's theorem due to Furstenberg and in particular the work of Host and Kra [H-K]. These ideas have potential to establish HLC for  $L \leq \mathbb{Z}^n$  of rank at least two (and non-degenerate), which would be quite remarkable.

(6) **Pythagorean Triples:** We turn to examples of orbits  $\mathcal{O}$  in  $\mathbb{Z}^n$  of groups acting by matrix multiplication rather than by translations (i.e. addition). By a Pythagorean triple we mean a point  $x \in \mathbb{Z}^3$  lying on the affine cone  $C$  given as  $\{x : F(x) = x_1^2 + x_2^2 - x_3^2 = 0\}$  and for which  $gcd(x_1, x_2, x_3) = 1$ . We are allowing  $x_j$  to be negative though in this example we could stick to all  $x_j > 0$ , so that such triples correspond exactly to primitive integral right triangles. Let  $O_F$  denote the orthogonal group of  $F$ , that is the set of  $3 \times 3$  matrices  $g$  for which  $F(xg) = F(x)$  for all  $x$ . Let  $O_F(\mathbb{Z})$  be the group of all such transformations with entries in  $\mathbb{Z}$ . Some elements of  $O_F(\mathbb{Z})$  are

$$A_1 = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}, A_3 = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

In fact  $O_F(\mathbb{Z})$  is generated by  $A_1, A_2$  and  $A_3$ . It is a big group and one can show that the set of all Pythagorean triples  $P$  is the orbit of  $(3, 4, 5)$  under  $O_F(\mathbb{Z})$ , i.e.  $P = (3, 4, 5)O_F(\mathbb{Z})$ . Following the lead of Dirichlet, let  $L$  be a subgroup of  $O_F(\mathbb{Z})$  and let  $\mathcal{O} = (3, 4, 5)L$  be the corresponding orbit of Pythagorean triples. The area  $A(x) = x_1 x_2 / 2$  of the corresponding triangle is in  $\mathbb{Q}[x_1, x_2, x_3]$ . We seek triangles in  $\mathcal{O}$  for which the area has few prime factors. What is the minimal divisibility of the areas of a Zariski dense (in  $Zcl(\mathcal{O})$ , which for us will be equal to  $C$ ) set of triples in  $\mathcal{O}$ ? We return to this later on. As a side comment, a similar problem asks which numbers are the square free parts of the areas of Pythagorean triangles in  $P$ ? This is the ancient "congruent number problem" about which much has been written especially because of its connection to the question of the ranks of a certain family of elliptic curves. Heegner [Hee] using his precious method for producing rational points on elliptic curves shows that any prime  $p \equiv 5$  or  $7 \pmod{8}$  is a congruent number. For a given such  $p$  the set of triangles realizing  $p$  is very sparse but never-the-less is Zariski dense in  $C$ . Via the same relation the congruent number problem is connected to automorphic  $L$ -functions through the Birch and Swinnerton-Dyer Conjecture (see [Wi]).

(7) **Integral Apollonian Packings:** As a final example before putting forth the general theory we discuss some Diophantine aspects of integral Apollonian packings. Descartes is well known among other things for his describing various geometric facts in terms of his Cartesian coordinates. One such example is the following relation between four mutually tangent circles:

\*Though the paper "Intersective polynomials and polynomial Szemerédi theorem" by V. Bergelson, A. Leibman and E. Lesigne posted on ArXiv Oct/25/07, begins to address this issue.



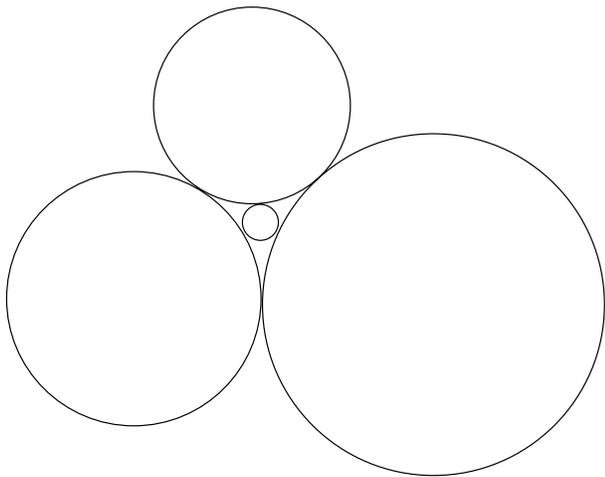
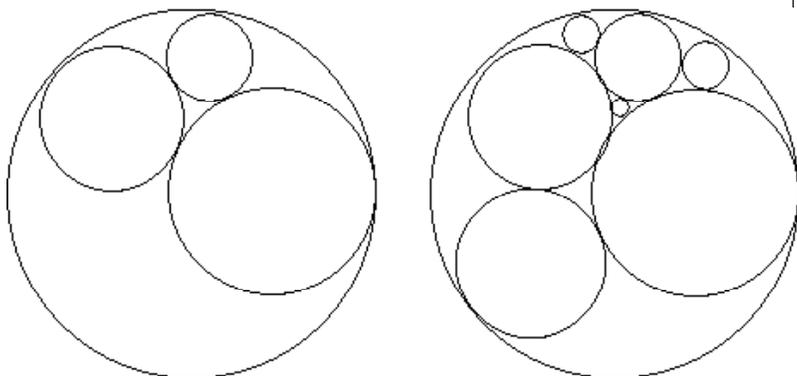


Figure 1.

If the radius of the  $j^{\text{th}}$  circle is  $R_j$ , then its curvature  $a_j$  is equal to  $1/R_j$ ,  $j = 1, 2, 3, 4$ . The relation is that

$$F(a_1, a_2, a_3, a_4) := 2(a_1^2 + a_2^2 + a_3^2 + a_4^2) - (a_1 + a_2 + a_3 + a_4)^2 = 0.$$

Consider now an Apollonian packing which is defined as follows; starting with 4 tangent circles of the first generation in Figure 2 (in this configuration the outer circle has all the other circles in its interior so by convention its curvature is  $-1/R$  where  $R$  is its radius).



Generation 1

Generation 2

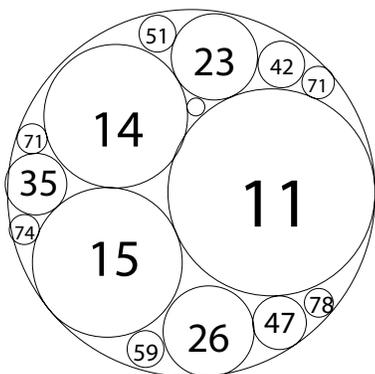


Figure 2.

Now place a circle in each of the 4 lune regions in generation 1 so that these are tangent to the three circles that bound the lune. The placement is possible and is unique according to a theorem of Apollonius. At generation 2, there are now 12 new lunes and we repeat the process ad-infinitum. The resulting packing by circles is called an Apollonian packing. The complement of all the open disks in the packing is a closed fractal set whose Hausdorff dimension  $\delta$  is approximately 1.30. Boyd [Bo] has shown that if  $N(T)$  is the number circles in the packing whose curvature is at most  $T$ , then  $\log N(T)/\log T \rightarrow \delta$  as  $T \rightarrow \infty$ . The interesting Diophantine fact is that if the initial 4 circles have integral curvatures then so do all the rest of the circles in the packing. This is apparent in the example in Figure 2 where the initial 4 circles have curvatures  $(-6, 11, 14, 23)$  and where the curvatures of each circle is displayed in the circle. It is customary in any lecture to offer at least one proof. Ours is the demonstration of this integrality of curvatures.

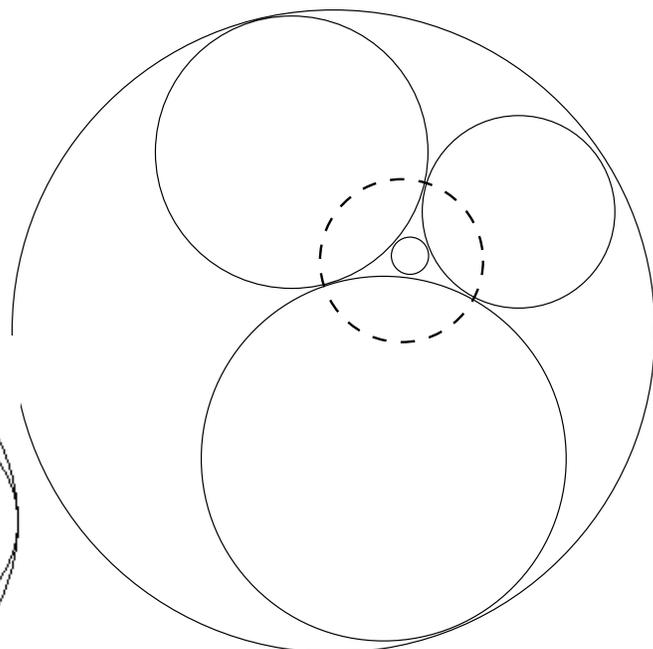


Figure 3.

In this figure the inversion  $S$  in the dotted circle, which is the unique circle orthogonal to the inside circles, takes the outermost circle to the innermost one and fixes the other three. It takes the 4-tuple  $(a_1, a_2, a_3, a_4)$  representing the curvatures of the 4 outer circles to  $(a'_1, a_2, a_3, a_4)$  where  $a'_1$  is the curvature of the inner most circle. From the Descartes relation it follows that  $a_1$  and  $a'_1$  are roots of the same quadratic equation and a simple calculation yields that  $a'_1 = -a_1 + 2a_2 + 2a_3 + 2a_4$ . This inversion is also the step which places the circle in the corresponding lune, that being a single step in the Apollonian packing. It follows that if the  $4 \times 4$  integral involutions  $S_1, S_2, S_3, S_4$  are given by

$$S_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}, S_2 = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}, S_3 = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix}, S_4 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

and  $A$  is the group generated by  $S_1, S_2, S_3, S_4$  then the orbit  $\mathcal{O} = (a_1, a_2, a_3, a_4)A$ , corresponds precisely to the configurations of 4 mutually tangent circles in the packing. Hence if  $a \in \mathbb{Z}^4$  and is primitive then so is every member of the orbit and in particular every curvature is an integer. The Diophantine properties of the numbers that are curvatures of an integral packing are quite subtle and are investigated in [G-L-M-W-Y]. The reason for the subtlety is that the Apollonian group  $A$  is clearly a subgroup of  $O_f(\mathbb{Z})$  but it is of infinite index in the latter (corresponding the dimension  $\delta = 1.30 \dots$ ). Still, the  $Zcl(A)$  is all of  $O_f$ , which is important for our investigation below. From the point of view of our theme in this lecture the immediate question is whether there are infinitely many circles in an integral packing with curvature a prime number. Or on looking at Figure 2, are there infinitely many “twin primes” that is pairs of tangent circles with curvatures that are both prime?

(8) Affine Orbits and Saturation: There is a simple and uniform formulation of all the questions above which is as follows: Let  $L$  be a group of morphisms (that is polynomial maps) of affine  $n$ -space which preserves  $\mathbb{Z}^n$ . Let  $c \in \mathbb{Z}^n$  and  $\mathcal{O} = cL$  the corresponding orbit. If  $f \in \mathbb{Q}[x_1, \dots, x_n]$  for which  $f(\mathcal{O})$  is integral and is infinite, we seek points  $x \in \mathcal{O}$  at which  $f(x)$  has few (or fewest) prime factors. Given such a pair  $(\mathcal{O}, f)$  set  $N = N(\mathcal{O}, f)$  the “conductor of the pair” to be the greatest common divisor of the numbers in  $f(\mathcal{O})$ . The key definition is the *saturation number*  $r_0(\mathcal{O}, f)$ , of the pair  $(\mathcal{O}, f)$ , which is the least  $r$  such the set of  $x \in \mathcal{O}$  for which  $f(x)/N$  has at most  $r$  prime factors, is Zariski dense in  $Zcl(\mathcal{O})$ . This number is by no means easy to determine and it is far from clear that it is even finite. Knowing it however answers all our questions. For example the following are easy to check

- (i)  $r_0(c + q\mathbb{Z}, x) = 1$  is Dirichlet’s Theorem.
- (ii)  $r_0(\mathbb{Z}, x(x+2)) = 2$  is the twin prime conjecture.
- (iii) If  $f \in \mathbb{Z}[x]$  and  $f$  factors into  $t$  irreducible factors over  $\mathbb{Q}[x]$ , then  $r_0(\mathbb{Z}, f) = t$  is equivalent to Schinzel’s hypothesis  $H$  [S-S] concerning simultaneous primality of  $t$  distinct irreducible integral polynomials in one variable.
- (iv) Let  $\mathcal{O} = c + L$  as in the HLC in (5). Then  $r_0(\mathcal{O}, x_1 x_2 \dots x_n) = n$  is equivalent to the HLC as stated in (5).

The fundamental general tool to study  $r_0$  is the Brun combinatorial sieve. He used his ingenious invention to show that  $r_0(\mathbb{Z}, x(x+2))$  is finite and his arguments can be easily extended to show that  $r_0(\mathbb{Z}, f) < \infty$  for any  $f \in \mathbb{Z}[x]$ . In fact the combinatorial sieve in any of its axiomatic modern formulations can be used to show that  $r_0(\mathcal{O}, f) < \infty$  for any orbit  $\mathcal{O}$  of  $L$  which is a subgroup of  $\mathbb{Z}^n$  acting by additive translations. As pointed out at the end of paragraph (2) above we insist on not restricting  $f(x)$  to be positive when looking for primes or numbers with few prime factors. The reason is that in this several variable context the condition that  $f(x) > 0, f \in \mathbb{Z}[x_1, \dots, x_n]$  can encode the general diophantine equation (for example if  $f(x) = 1 - g^2(x)$  then  $f(x) > 0$  is equivalent to  $g(x) = 0$ ). The work of Matiyasevich et al [Ma] on Hilbert’s 10<sup>th</sup> problem shows that given any recursively enumerable subset  $S$  of the positive integers, there is a  $g \in \mathbb{Z}[x_1, \dots, x_{10}]$  such that  $S$  is exactly the set of positive values assumed by  $g$ . From this it is straight forward to construct an  $f \in \mathbb{Z}[x_1, \dots, x_{10}]$  such that for any  $r < \infty, \{x \in \mathbb{Z}^{10} : f(x) > 0 \text{ and } f(x) \text{ is a product of at most } r \text{ primes}\}$  is not Zariski dense in  $Zcl\{x \in \mathbb{Z}^{10} : f(x) > 0\}$ . That is if we insist on positive values for  $f$  we may lose the basic finiteness of saturation property.

Returning to one variable the theory of the sieve has been developed and refined in far-reaching ways to give good bounds for  $r_0$ . For example

$$r_0(\mathbb{Z}, x(x+2)) \leq 3 \quad (\text{Chen 1973})$$

$$r_0(\mathbb{Z}, x^2 + 1) \leq 2 \quad (\text{Iwaniec 1978})$$

$$r_0(\mathbb{Z}, f) \leq d + 1, \text{ if } f \text{ is irreducible over } \mathbb{Q}[x] \text{ and has degree } d \text{ [H-R].}$$

The first two are especially striking as they come as close as possible to the twin prime and Euler problems, without solving them.

While there are interesting examples of groups  $L$  acting nonlinearly and morphically on  $A^n$  and preserving  $\mathbb{Z}^n$ , that come from the actions of mapping class groups on representation varieties [Go], the understanding of anything about saturation numbers in such cases is very difficult and is at its infancy. For  $L$  acting linearly (as in paragraphs (6) and (7)) a theory can be developed.

(9) An Affine Linear Sieve: The classical setting is concerned with motions of  $n$ -space of the form  $x \rightarrow x + b$ . In this affine linear setting we allow multiplication as well, that is transformations of the form  $x \rightarrow xa + b$  with  $a \in GL_n(\mathbb{Z})$  and  $b \in \mathbb{Z}^n$  (such as the orthogonal group examples (6) and (7)). Note that it is only for  $n \geq 2$  that this group of motions is significantly larger than translations (since  $GL_1(\mathbb{Z}) = \pm 1$ ). For the purpose of developing a Brun combinatorial sieve, apparently multiplication is quite a bit more difficult than addition. The basic problems for our pair  $(\mathcal{O}, f)$  are

- (i) Is  $r_0(\mathcal{O}, f)$  finite?
- (ii) If it is, then to give good upper bounds for  $r_0(\mathcal{O}, f)$ . Ideally these should be in terms of the degree of  $f$  and its factorization in the coordinate ring of  $Zcl(\mathcal{O})$ , as has been done in the setting of one variable [H-R].
- (iii) To determine  $r_0(\mathcal{O}, f)$  for some interesting pairs and to give an algorithm to predict its exact value in general, that is a generalized local to global conjecture for which HLC and Schinzel’s Hypothesis  $H_t$  are special cases.

When  $L$  is a group of affine linear transformations we now have a theory that comes close to answering these questions, there being the caveat of tori (see below) and some other nontrivial technical issues that still need to be resolved in general. In Bourgain-Gamburd-Sarnak ([B-G-S1], [B-G-S2]) the finiteness of  $r_0(\mathcal{O}, f)$  is proven in many cases. The new tools needed to address these questions, as well as the general setup that we have been discussing are introduced in these papers. The proof given there of the finiteness does not yield any feasible values for  $r_0(\mathcal{O}, f)$ . In [N-S] the problem is studied in the case that  $L$  is a congruence subgroup of the  $\mathbb{Q}$  points of a semi-simple linear algebraic group defined over  $\mathbb{Q}$ , such as the group  $O_f(\mathbb{Z})$  in paragraphs (6) and (7) above (an affine linear action can be linearized by doubling the number of variables). For such congruence  $L$ ’s we develop the combinatorial sieve using tools from the general theory of automorphic forms on such groups and in particular make use of the strong bounds towards the general Ramanujan Conjectures that are now known ([Sa1], [Cl]). With this we get effective bounds for  $r_0(\mathcal{O}, f)$  which in many such cases are of the same quality as what is known in one variable.

There is a lacuna in this affine linear sieve theory coming from tori. As we mentioned allowing multiplication as well as addition, is what makes the problem hard and in fact pure multiplication is simply too hard and even the finiteness is questionable in that case. Consider the example of  $L = \left\{ \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}^n : n \in \mathbb{Z} \right\} \leq SL_2(\mathbb{Z})$ .  $L$  is infinite cyclic,  $Zcl(L)$  is a torus and if  $\mathcal{O} = (1, 0) \cdot L$  then  $Zcl(\mathcal{O})$  is the hyperbola  $\{(x_1, x_2) = x_1^2 - 3x_1x_2 + x_2^2 = 1\}$ . The orbit consists of pairs  $(F_{2n}, F_{2n-2})$   $n \in \mathbb{Z}$  where  $F_m$  is the  $m$ <sup>th</sup> Fibonacci number. This kind of sequence is too sparse both from the analytic and algebraic points of view to do any kind of (finite) sieve. While it is conjectured that  $F_m$  is prime for infinitely many  $m$ , as was pointed out to me



by Lagarias, standard heuristic probabilistic considerations suggest a very different behavior for  $F_{2n}$ . Indeed  $F_{2n} = F_n \cdot L_n$  where  $L_n$  is the  $n^{\text{th}}$  Lucas number and assuming a probabilistic model for the number of prime factors of a large integer in terms of its size and that  $F_n$  and  $L_n$  are independent leads to  $F_{2n}$  having an unbounded number of prime factors as  $n \rightarrow \infty$ . A precise conjecture along these lines is put forth in [B-L-M-S] (see Conjecture 5.1). In our language this asserts that if  $\mathcal{O}$  is as above and  $f(x_1, x_2) = x_1$  then  $r_0(\mathcal{O}, f) = \infty$ . It would be very interesting to produce an example of a pair  $(\mathcal{O}, f)$  for which one can prove that  $r_0(\mathcal{O}, f)$  is infinite. In view of the above we must steer clear of tori and the precise setting in which the affine linear sieve is developed (see [Sa-Sa]) is for linear  $L$ 's for which the radical (the largest normal solvable subgroup) of the  $\mathbb{Q}$  linear algebraic group  $G := \text{Zcl}(L)$ , contains no tori (the unipotent radical causes no difficulties).

Applying this theory to the examples of orthogonal groups in (6) and (7) we obtain the following. Let  $F(x_1, x_2, x_3) = x_1^2 + x_2^2 - x_3^2$  and  $L \leq O_F(\mathbb{Z})$ . Assume that  $L$  is not an elementary group (in particular not finite or abelian, in fact precisely that  $\text{Zcl}(L)$  is either of the linear algebraic groups  $O_F$  or  $SO_F$ ). If  $\mathcal{O} = (3, 4, 5)L$ , then  $\text{Zcl}(\mathcal{O}) = C$  the affine cone;  $F = 0$ . For  $f \in \mathbb{Z}[x_1, x_2, x_3]$  the results in [B-G-S2] imply that  $r_0(\mathcal{O}, f) < \infty$ . In particular this applies to  $f(x) = A(x) = x_1 x_2 / 2$ , the area. This says that given such an orbit of Pythagorean triangles (which may be very sparse!) there is an  $r < \infty$  such that the set of triangles in  $\mathcal{O}$  whose areas have at most  $r$  prime factors is Zariski dense in  $C$ . It is elementary that  $N = N(\mathcal{O}, A) = 6$ . From the ancient parametrization of all the Pythagorean triples  $P$  (i.e. the  $\mathbb{Q}$  morphism of  $A^2$  into  $C$ ) these are all of the form  $(x_1, x_2, x_3) = (a^2 - b^2, 2ab, a^2 + b^2)$  with  $a, b \in \mathbb{Z}$ ,  $(a, b) = 1$  and not both odd, one sees that  $A/6 = (a - b)(a + b)(ab)/6$ . Now the last has at most two prime factors for only finitely many pairs  $(a, b)$ . The set of  $(a, b)$  for which it has at most 3 prime factors lie in a finite union of curves in  $C$  (and if HLC is true for  $\mathcal{O} = (2, 2, 0) + (3, 3, 1)\mathbb{Z}$ , i.e. this rank one orbit in  $\mathbb{Z}^3$ , then these curves contain infinitely many points with  $A/6$  having 3 prime factors). Hence for any  $\mathcal{O}$  as above  $r_0(\mathcal{O}, A) \geq 4$ . The general local to global conjectures [B-G-S2] then assert that  $r_0(\mathcal{O}, A) = 4$  for any such orbit. Interestingly the recent advance in [G-T2] mentioned in (5) above just suffices to prove that for the full set of Pythagorean triples  $P$ ,  $r_0(P, A) = 4$ . Put another way the minimal divisibility of the areas of a Zariski dense set of Pythagorean triangles is 6 (here we include the forced factors 3 and 2). The deduction is immediate, set  $a = 2x$  and  $b = 3y$  in the ancient parametrization. Then  $A/6 = xy(2x + 3y)(2x - 3y)$  and apply [G-T2] to  $\mathcal{O} = L = (1, 0, 2, 2)\mathbb{Z} + (0, 1, 3, -3)\mathbb{Z}$ . For some other applications of [G-T1] see Granville [Gr].

As an example of an application of the affine linear sieve in the context of an  $L$  which is a congruence group, consider an integral quadratic form  $F(x)$  in 3-variables. That is  $F(x) = x'Ax$  where  $A$  is  $3 \times 3$  symmetric and is integral on the diagonal and half integral on the off-diagonal. We assume that  $F$  is indefinite over the reals but that it is anisotropic over  $\mathbb{Q}$  (so  $F(x) = 0$  for  $x \in \mathbb{Z}^3$  implies that  $x = 0$ ) and that  $\det A$  is square free (so

$F(x_1, x_2, x_3) = x_1^2 + x_2^2 - 7x_3^2$  is an example, the anisotropy following from looking at  $F(x) \equiv 0 \pmod{8}$ ). Let  $0 \neq t \in \mathbb{Z}$  for which  $V_t(\mathbb{Z}) = \{x \in \mathbb{Z}^3 : F(x) = t\}$  is nonempty, which according to the work of Hasse and Siegel will happen iff there are no local congruence obstructions to solving  $F(x) \equiv t \pmod{q}$  for  $q \geq 1$ . In this case  $V_t(\mathbb{Z})$  is a finite union of  $O_F(\mathbb{Z})$  orbits and  $\text{Zcl}(V_t(\mathbb{Z})) = V_t$ , the affine quadric  $\{x : F(x) = t\}$ . We seek points in  $V_t(\mathbb{Z})$  whose coordinates have few prime factors, i.e. to estimate  $r_0(V_t(\mathbb{Z}), x_1 x_2 x_3)$ . By the general finiteness theorem,  $r_0(V_t(\mathbb{Z}), x_1 x_2 x_3)$  is finite. However by developing optimal weighted counting results on such quadrics and also exploiting the best bounds known towards the Ramanujan-Selberg Conjecture, it is shown in [L-S] that  $r_0(V_t(\mathbb{Z}), x_1 x_2 x_3) \leq 26$ .

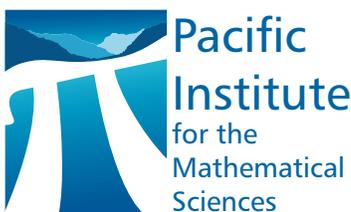
We turn to the Apollonian packing. An extension of the  $(\mathcal{O}, f)$  finiteness theorem in [B-G-S2] applies to the orbit  $\mathcal{O} = aL$  for any  $L$  which is Zariski dense in  $O_F$ , where  $F$  is the quadratic form in 4-variables in paragraph (7). In particular it applies to the Apollonian group  $A$  with  $f(x) = x_1 x_2 x_3 x_4$ . This asserts that in any given integral packing there is an  $r < \infty$  such that the set of 4 mutually tangent circles in the packing for which all 4 curvatures have at most  $r$  prime factors is Zariski dense in  $\text{Zcl}(\mathcal{O}) = C = \{x : F(x) = 0\}$ . One can determine  $r_0$  for  $\mathcal{O} = a \cdot A$  and some special  $f$ 's using some ad hoc and elementary methods together with (ii) of paragraph (4). In [Sa3] it is shown that  $r_0(\mathcal{O}, x_1) = 1$  and  $r_0(\mathcal{O}, x_1 x_2) = 2$ , from which it follows that in any such packing there are infinitely many circles whose curvatures are prime and better still there are infinitely many pairs of tangent circles both of whose curvatures are prime.

As a final example of an interesting pair  $(\mathcal{O}, f)$  for which we can determine  $r_0$ , consider the variety  $V_t$  in affine  $n^2$ -space given by  $V_t = \left\{ X = (x_{ij})_{\substack{i=1,\dots,n \\ j=1,\dots,n}} : \det X = t \right\}$ . For  $t$  a nonzero integer  $V_t(\mathbb{Z})$  consists of a finite union of  $L = SL_n(\mathbb{Z})$  orbits where the action of  $g$  is by  $X \rightarrow X \cdot g$ . In [N-S] we show using Vinogradov's methods mentioned in (5), that if  $n \geq 3$  then  $r_0(V_t(\mathbb{Z}), \prod_{i,j} x_{ij}) = n^2$ .

In particular, examining the conductor  $N = N(V_t(\mathbb{Z}), \prod_{i,j} x_{ij})$  in detail we deduce that the set of  $n \times n$  integral matrices of determinant  $t$  all of whose entries are prime, is Zariski dense in  $V_t$  iff  $t \equiv 0 \pmod{2^{n-1}}$ . This should of course, also hold for  $n = 2$  where it is concerned with the equation  $x_{11}x_{22} - x_{12}x_{21} = t$  and the  $x_{ij}$ 's are to be primes. The best that appears to be known concerning this is the recent development by [G-G-P-Y] from which it follows that for this  $n=2$  case,  $r_0(V_t(\mathbb{Z}), x_{11}x_{12}x_{21}x_{22}) = 4$ , for at least one  $t$  in  $\{2, 4, 6\}$ .

(10) **Comments about Proofs:** I end the lecture with a very brief hint as to what is involved in developing a combinatorial sieve in the affine linear context. This entails getting a little more technical. Let  $\mathcal{O} = cL$  be our orbit and  $f \in \mathbb{Z}[x_1, \dots, x_n]$ . After some algebro-geometric reductions of the problems (using the  $\mathbb{Q}$  dominant morphisms from  $G = \text{Zcl}(L)$  to  $V = \text{Zcl}(\mathcal{O})$  and  $\tilde{G}$  to  $G$  where  $\tilde{G}$  is the simply connected cover of  $G$ ) we can assume that  $\mathcal{O}$  is the group  $L$  itself (as a group of matrices in the affine space of

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$n \times n$  matrices) and  $Zcl(\mathcal{O}) = Zcl(L) = G$  is a simply connected  $\mathbb{Q}$ -group. To do any kind of sieving we need to order the elements of  $L$  so as to carry out some truncated inclusion-exclusion procedure, this being at the heart of Brun's method. Usually one orders by archimedean size perhaps with positive weights, however in this general setting we don't know how to do this, so we order  $L$  combinatorially instead. For the groups that we are considering and for the purpose of proving that  $r_0(\mathcal{O}, f)$  is finite, we can (according to a theorem of Tits) assume that  $L$  is free on two generators  $A$  and  $B$ . We use the tree structure of the Cayley graph  $T = (L, S)$  of  $L$  with respect to the generators  $S = \{A, A^{-1}, B, B^{-1}\}$ .  $T$  is a 4-regular tree;

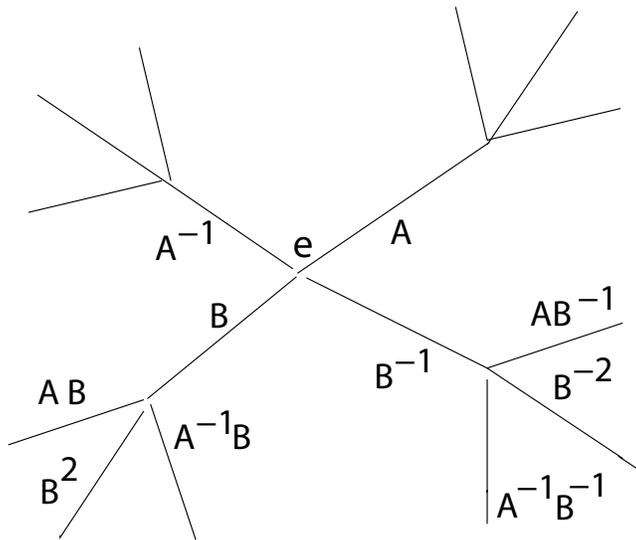


Figure 4.

For  $x, y \in T$  let  $d(x, y)$  denote the distance from  $x$  to  $y$  in the tree. The key sums that arise in sieving on  $L$  for divisibility of  $f$  are:

For  $d \geq 1$  square-free and  $x_0 \in T$ ,

$$S(Y, d) := \sum_{\substack{x \in L \\ d(x, x_0) \leq Y \\ f(x) \equiv 0(d)}} 1,$$

or perhaps with 1 replaced by positive weights.

We are interested in  $S(Y, d)$  when  $Y$  is large and  $d$  as large as  $e^{\alpha Y}$  for  $\alpha > 0$ . The larger the  $\alpha$  for which  $S$  can be understood the better. To study such sums a couple of key features intervene:

(i) Algebraic stabilization: This is the analogue of the Chinese remainder theorem. We state it for the basic case of  $G = SL_n$ , it is valid for  $G$  semisimple and simply connected. It is due (originally) to Matthews-Vaserstein and Weisfeiler [M-V-W] who employ the classification of finite simple groups in the proof. Let  $L \leq SL_n(\mathbb{Z})$  be Zariski dense in  $SL_n$ . Then there is a positive integer  $\nu = \nu(L)$  such that for  $d$  with  $(d, \nu) = 1$  the reduction  $L \rightarrow SL_n(\mathbb{Z}/d\mathbb{Z})$  is onto.

This eventually allows us to bring in more standard tools from arithmetic algebraic geometry, in order to identify the main term in the form

$$S(Y, d) = \beta(d) S(Y, 1) + R(Y, d).$$

Here  $\beta(d)$  is a multiplicative arithmetical function associated with counting points mod  $d$  on the variety  $G \cap \{f=0\}$  and  $R$  is the remainder which is expected to be smaller. The demonstration of

the latter for the purpose of sieving far enough to get the finiteness of  $r_0(\mathcal{O}, f)$ , leads to the second feature.

(ii) The (finite) Cayley graphs  $(SL_n(\mathbb{Z}/d\mathbb{Z}), S)$  are an expander family as  $d \rightarrow \infty$  (see [Sa] for a definition of expanders and [Lub] where this is conjectured). As yet, this expander property has not been established in general and this is the main reason that the finiteness of  $r_0(\mathcal{O}, f)$  has not been established in general for the affine linear sieve. It is proven for  $SL_2$  and related groups for  $d$  square free, in [B-G-S2]. The proof uses a variety of inputs some of which were to me at least, quite unexpected. We list them for the simpler case that  $d = p$  is prime:

(a) The dichotomy that an irreducible complex representation of  $G(\mathbb{Z}/p\mathbb{Z})$  is either 1-dimensional or is of very large dimension (here  $p \rightarrow \infty$ ) coupled with a "softer" upper bound density theorem for multiplicities of exceptional eigenvalues of the Cayley graphs, leads to a proof of the key spectral gap defining an expander [S-X]. For the soft upper bound we use techniques from arithmetic combinatorics.

(b) Sum-Product Theorem [B-K-T]: This is an elementary and very useful theorem concerning mixing the additive and multiplicative structures of a finite field. Let  $\epsilon > 0$  be given, there is a  $\delta > 0$ ,  $\delta = \delta(\epsilon)$ , such that if  $A \subset \mathbb{F}_p$  and  $|A| \leq p^{1-\epsilon}$  then  $|A + A| + |A \cdot A| \geq |A|^{1+\delta}$  (here  $p$  is sufficiently large).

(c) Helfgott's  $SL_2(\mathbb{F}_p)$  Theorem [He]: Let  $\epsilon > 0$  there is  $\delta = \delta(\epsilon) > 0$  such that if  $A \subset SL_2(\mathbb{F}_p)$ ,  $A$  is not contained in a proper subgroup of  $SL_2(\mathbb{F}_p)$  and  $|A| \leq |SL_2(\mathbb{F}_p)|^{1-\epsilon}$ , then  $|A \cdot A \cdot A| \geq |A|^{1+\delta}$ .

(d) Balog-Szemerédi, Gowers Theorem: This is a purely combinatorial theorem from graph theory which is used in [B-G] to give the required upper bounds on counting closed circuits in the graph, and leads to a proof that  $(SL_2(\mathbb{Z}/p\mathbb{Z}), S)$  is an expander family.

A point worth noting is that once the affine sieve is set up and gives lower bounds in our combinatorial group theoretic ordering, for points in  $\mathcal{O}$  for which  $f$  has at most  $r$  prime factors, the expander property is used again and in a different way to demonstrate the Zariski density of these points.

To end let me highlight the fundamental difference between the additive translational counting and the affine linear counting which necessitates the introduction of expanders. In  $\mathbb{Z}$  the boundary of a large interval is small compared with the size of the interval and the same is true uniformly for an arithmetic progression of common difference  $q$  in the interval, even for  $q$  almost as large as the interval length. On the other hand on a  $k$ -regular tree ( $k \geq 3$ ) this is not true. Given a big ball  $B$  (or any large finite set), the size of the boundary  $\partial B$  is of the same order of magnitude as  $B$ . It is exactly the expander property that allows one to draw an effective approximation for the number of points in  $B$  lying in the orbit with a congruence condition.

*Acknowledgements:* In developing this theory of an affine sieve and the geometric view point described in this lecture, I have benefited from discussions with many people. First and foremost with my collaborators on the different aspects of the theory, Bourgain and Gamburd, Liu, Nevo and Salehi. Also with Lubotzky, Katz, Lindenstrauss and Mazur. Finally thanks to Lagarias who pointed me to his joint works on integral Apollonian packings and their subtle diophantine features.

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# Call for PIMS Proposals 2009

The **Pacific Institute for the Mathematical Sciences** welcomes applications for support for conferences, workshops, seminars and related activities in the Mathematical Sciences, to occur after April 1, 2009. PIMS also invites scientists at PIMS universities to submit letters of intent for a Period of Concentration of a PIMS Collaborative Research Group (CRG).

## Deadline

Proposals must be received by the deadline of October 1, 2008. The Scientific Review Panel will review proposals in November, 2008, and results will be announced by January 31, 2009. For submission rules and requirements, please see <http://www.pims.math.ca>.

All applications will receive a confirmation within one week. Enquiries regarding the status of proposals should be directed to [proposal@pims.math.ca](mailto:proposal@pims.math.ca). Incomplete proposals and late submissions may not be included in the scientific review process.

## Content of Proposals - General Activities

All proposals should contain the sections below, and give affiliations of all persons cited. The summary of objectives should be at most 100 words and may appear in public announcements. Further details may be discussed in section 8, as well as any unusual features of the proposal.

1. Title of proposal.
2. Name, affiliation and CV/biosketch of main organizer(s).
3. Place and dates.
4. Amount of funding requested from PIMS.
5. Summary of scientific and other objectives.
6. List of participants (invited, confirmed, etc.).
7. Intended audience, provision for students.
8. General comments.
9. Budget and list of other funding and support sources.



# PIMS Welcomes New Members to Board and Scientific Review Panel

## PIMS Board of Directors

**F**ernando Aguilar graduated as a Civil Engineer (M.S.) in Hydraulic Resources Management in 1981, and received his MBA in 1990 from the Universidad of Los Andes in Bogota, Colombia. He completed the Stanford Executive program at Stanford University in 2003, and the Directors Education Program at the University of Calgary in 2007.



With 26 years of experience on all continents, Mr. Aguilar has held a variety of positions in diversified conglomerates of Schlumberger in various technology, business and oilfield sectors specializing in merging cultures, restructuring and consolidating companies for enhanced results and improved shareholder return. Mr. Aguilar is currently Executive Vice-President for Canada Land Processing, Canada Land Library, and Western Hemisphere Land Acquisition with CGGVeritas. In April, 2008, he will assume the new position of President of the Eastern Hemisphere of the corporation. He has been a member of the PIMS Board since August, 2007.

**K**atherine Bergman is the Dean of Science at the University of Regina. Dr. Bergman graduated from the University of Waterloo with a B.Sc. in Biology (Honours). She received her M.Sc. in Geology (Palaeontology) and her Ph.D. in Geology (Sedimentology) from McMaster University.



Dr. Bergman has been at the U.Regina since 1992 and has been the recipient of numerous Natural Sciences and Engineering Research Council of Canada (NSERC) awards. She has written and presented extensively on geological issues. She was appointed U.Regina Dean of Science in 2001. She has been on the PIMS Board since December, 2007.

**C**harmaine Dean is Professor and Burnaby Mountain Research Chair in the Department of Statistics and Actuarial Science at Simon Fraser University. Her research interest lies in the development of methodology for the analysis of correlated count data common in longitudinal and spatio-temporal analyses. Much of this work has been motivated by direct applications to important practical problems in biostatistics and ecology.



Dr. Dean is Past-President of the Statistical Society of Canada, was 2002 President of the International Biometrics Society, Western North American Region, served as President of the Biostatistics Section of the Statistical Society of Canada, and has given six years of service to the Natural Sciences and Engineering Research Council of Canada, including two as President of the Statistical Sciences Grant Selection Committee. She has been a member of the PIMS Board since July, 2007.

**D**arrell Duffie holds the Dean Witter Distinguished Professorship in Finance at the Graduate School of Business, Stanford University, where he has been a member of the finance faculty since receiving his PhD at Stanford in 1984. He received his Masters of Economics from the University of New England (Australia), and his B.Sc. from the University of New Brunswick.

Dr. Duffie is the author of *Dynamic Asset Pricing Theory* (Princeton University Press, third edition 2001) and coauthor with Ken Singleton of *Credit Risk* (Princeton University Press, 2004). His recent research focuses on asset pricing, credit risk, fixed-income securities, and over-the-counter markets. He is currently on the editorial boards of *Econometrica* and the *Journal of Financial Economics*, among other journals. His term as a PIMS Board Member will begin on July 1, 2008.

**R**ose Goldstein is the Vice-President, Research, at the University of Calgary, beginning July 1, 2007. Dr. Goldstein takes over the position held by Dennis Salahub.



Prior to her appointment at U.Calgary, Dr. Goldstein was Professor and Vice-Dean, Academic Affairs, in the University of Ottawa's Faculty of Medicine. Dr. Goldstein is also a practicing rheumatologist (a medical doctor who specializes in the treatment of joint and connective tissue-related diseases such as rheumatoid arthritis, osteoarthritis, fibromyalgia and lupus). She received her B.Sc. and M.D. degrees from McGill University. She has been on the PIMS Board of Directors since July, 2007.

**R**ichard Keeler is the Associate Vice-President, Research, at the University of Victoria, responsible for Research Services as part of the Office of Research. He has a B.Sc. in Physics from McGill University, and an M.Sc. and Ph.D. from the University of British Columbia.



As an NSERC postdoctoral fellow, he was part of a team at the CERN Laboratory in Geneva Switzerland that discovered two fundamental particles. An appointment (1983) as an NSERC University Research Fellow and professor at the University of Victoria followed.

His research interests focus on electroweak interactions in particle physics - the process that powers the sun and governs atomic decay. Presently he is working on the ATLAS project at the Large Hadron Collider under construction at CERN. The goal is to answer the question, "why do particles have mass?" He has been on the PIMS Board of Directors since June, 2007.

# Board of Directors Panel

## Scientific Review Panel

**G**unnar Carlsson is the Anne & Bill Swindells Professor in the Department of Mathematics at Stanford University. His research interests lie in algebraic topology, algebraic K-theory and number theory.

Dr. Carlsson received his Ph.D. from Stanford University. He was a professor at the University of California, San Diego, and at Princeton University, before arriving at Stanford in 1991. He has been a member of the PIMS SRP since 2007.



**W**alter Craig received his doctorate from the Courant Institute in 1981, with PhD advisor L. Nirenberg, after an undergraduate degree from Berkeley. He has held faculty and research positions in the mathematics departments at the California Institute of Technology, Stanford University and Brown University, before moving to McMaster University as the Canada Research Chair of Mathematical Analysis and its Applications in 2000.



His research interests are in nonlinear partial differential equations and dynamical systems, with a focus on problems stemming from classical mechanics, fluid dynamics, and quantum mechanics. He has worked on the problem of free surface water waves, on KAM theory for partial differential equations and other systems with infinitely many degrees of freedom, on the propagation of singularities for Schrodinger's equations, on the singular set of solutions of the Navier-Stokes equations, and on the general theory of Hamiltonian partial differential equations. He is particularly interested in research in which surprising connections are uncovered between seemingly disparate parts of mathematics, as well as in situations in which theoretical results in mathematical analysis influence experimental or numerical approaches to a physical problem, and vice versa.

Dr. Craig is a Fellow of the Fields Institute and of the Royal Society of Canada, as well as having been a Sloan Research Fellow, a Bantrell Fellow and a NSF Presidential Young Investigator. He has served on the Scientific Advisory Panel of the Fields Institute, the Comité Consultatif of the Centre de Recherches Mathématiques, on the AMS Council and Executive Committees, and he is currently serving on a number of editorial boards of mathematics journals. He has been a member of the PIMS SRP since 2007.

**B**ruce Reed received his doctorate from McGill University in 1986. He has been a faculty member at the University of Waterloo and Carnegie Mellon University and a charge de recherche and directeur de recherche of the CNRS in France. He holds a Canada Research Chair in Graph Theory at McGill University.

Professor Reed's research interests lie at the intersection of computer science and mathematics. He is particularly interested in graph theory and discrete stochastic processes. He has given invited talks around the world, including at the 2002 ICM in Beijing. He has been a member of the PIMS SRP since 2007.

## PIMS Postdoctoral Fellows for 2008-2009

PIMS is pleased to announce the PIMS Postdoctoral Fellows (PDFs) for 2008-2009. The members of the review panel are:

Berndt Brenken (U.Calgary), Valentine Kabanets (SFU), Joanna Karczmarek (UBC), Stephen Kirkland (U.Regina), Mary-Catherine Kropinski (SFU), Arturo Pianzola (U.Alberta), Anthony Quas (U.Victoria), Paul Tseng (U.Washington)

### 2008-2009 PIMS Postdoctoral Fellows

#### Simon Fraser University

**Yves van Gennip:** CRG PDF in Partial Differential Equations, *Sponsor: Rustum Choksi*

**Ariel Gabizon:** Computer Science, *Sponsor: Valentine Kabanets*

**Katherine Stange:** Number Theory, *Sponsor: Nils Bruin*

#### University of Alberta

**Rajendran Prakash:** CRG PDF in Harmonic analysis, *Sponsor: Anthony To-Ming Lau*

**Kaneenika Sinha:** Number Theory, *Sponsor: Matilde Lalin*

#### University of British Columbia

**Francois Caron:** CRG PDF in Bayesian modeling and computation for networks, *Sponsor: Raphael Gottardo*

**Mohammad El Smaily:** CRG PDF in Partial Differential Equations, *Sponsor: Nassif Ghoussoub*

**Simon Bonner:** Statistics, *Sponsor: Nancy Heckman*

#### University of Calgary

**Illia Karabash:** Partial Differential Equations, *Sponsor: Paul Binding*

**Dilian Yang:** Partial Differential Equations, *Sponsor: Berndt Brenken*

#### University of Regina

**Ebrahim Samei:** Harmonic Analysis, *Sponsor: Douglas Farenick*

#### University of Victoria

**Ian Ross:** CRG PDF in Mathematical Problems in Climate Modelling, *Sponsor: Adam Monahan*

**Chan-Ho Suh:** Topology, *Sponsor: Ryan Budney*

#### University of Washington

**Hongyu Liu:** Inverse Problems, *Sponsor: Gunther Uhlmann*

**Clement Pernet:** Computational Algebra, *Sponsor: William Stein*

**Thordis Thorarinsdottir:** Statistics, *Sponsor: Tilmann Gneiting*

# PIMS Scientists Receive Honours

## Simon Fraser University

**Malgorzata Dubiel** (Simon Fraser University, PIMS SFU Education Coordinator) has been awarded a 3M Teaching Fellowship for 2008.

The award, now in its 23<sup>rd</sup> year, was established by 3M Canada in collaboration with the Society of Teaching and Learning in Higher Education. It is generally regarded as the highest honour given in Canada to recognize excellence in teaching.



## University of Alberta

The CAIMS\*SCMAI Research Prize 2007 has been awarded to **Gordon Swaters** (University of Alberta). The prize recognizes innovative and exceptional research contributions in an emerging area of applied or industrial mathematics.

Professor Swaters, whose research is focused on understanding the dynamics of ocean currents, has received the CAIMS\*SCMAI Research Prize from the Canadian Applied and Industrial Mathematics Society and the President's Prize from the Canadian Meteorological and Oceanographic Society. Within the University of Alberta, Dr. Swaters is recognized as a member of one of sixteen "Areas of Established Research Excellence" and has received the Faculty of Science Research Award, the McCalla Research Professorship and the Killam Annual Professorship.

## University of British Columbia

**Martin Barlow** is the recipient of the CMS 2008 Jeffery-Williams Prize, which recognizes mathematicians who have made outstanding contributions to mathematical research.

Dr. Barlow is the leading international expert in the study of diffusions on fractals and other disordered media. He has made a number of profound contributions to a variety of fields including probabilistic methods in partial differential equations, stochastic differential equations, filtration enlargement, local times, measure-valued diffusions and mathematical finance.

Past distinctions for Dr. Barlow include the Rollo Davidson Prize from Cambridge University, the Junior Whitehead Prize from the London Mathematical Society and an invited lecture at the 1990 ICM in Kyoto. He has served the Canadian mathematical community on the Research Committee of the CMS and on the Editorial Board of the Canadian Journal Mathematics and the Canadian Mathematical Bulletin. He also has served on a number of international panels and editorial boards and recently finished a term as Editor-in-Chief of Electronic Communications in Probability. He is a Fellow of the Royal Society of Canada and in 2006 was elected Fellow of the Royal Society (London).

Dr. Barlow will present the 2008 Jeffery-Williams Prize Lecture at the CMS Summer Meeting in Montréal, in June, 2008.

**David Brydges** has been elected as a Fellow of the Royal Society of Canada.

The RSC founded in 1882, is Canada's oldest and most prestigious scholarly organization, recognizing "the extraordinary accomplishments of persons of talent, expertise and creativity in all fields."

**Paul Gustafson** has been awarded the 2008 CRM-SSC award.

**Izabella Laba** has been awarded the CMS 2008 Krieger-Nelson Prize, which recognizes outstanding research by a female mathematician.

Dr. Laba has established a position as one of Canada's leading harmonic analysts. She has made major contributions to the

Takeya problem, and to the study of translational tilings and distance sets. Her outstanding work has been recognized with a UBC Faculty of Science Achievement Award for Research in 2002 and the CMS Coxeter-James Prize in 2004. She is one of the lead organizers of the thematic program "New trends in harmonic analysis", which is being held at the Fields Institute from January to June 2008.

Dr. Laba will present the 2008 Krieger-Nelson Prize Lecture at the CMS Summer Meeting in Montreal in June, 2008.

**Ed Perkins** (PIMS Board of Directors) has received one of 10 Canada Council Killam Research Fellowships, one of Canada's most distinguished annual awards. Dr. Perkins received the award for his research work, "Interactive Branching Population Models".

Killam Research Fellowships enable Canada's best scientists and scholars to devote two years to full-time research. Made possible by a bequest of Mrs. Dorothy J. Killam, the awards support scholars engaged in research projects of outstanding merit in the humanities, social sciences, natural sciences, health sciences, engineering and interdisciplinary studies within these fields.

Dr. Perkins was also appointed as a Fellow to the Royal Society (London) in 2007. Fellows are elected for their contributions to science, both in fundamental research resulting in greater understanding, and also in leading and directing scientific and technological progress in industry and research establishments.

On Nov. 9, 2007, **Dale Rolfsen** was awarded an honorary doctorate (Docteur Honoris Causa) by the University of Caen, France, on Nov. 9, 2007.



Dr. Rolfsen receives his honorary doctorate at the University of Caen.

# PIMS Scientists Receive Honours

## University of Victoria

## University of Washington

**Pauline van den Driessche** has been awarded the CMS 2007 Krieger-Nelson Prize. The Krieger-Nelson Prize recognizes outstanding research by a female mathematician.

Dr. van den Driessche presented the 2007 Krieger-Nelson Prize Lecture at the CMS Summer Meeting hosted by the University of Manitoba in June, 2007.

Dr. van den Driessche was also awarded the first Olga Taussky Todd Lecture Award. Dr. van den Driessche presented her lecture at the International Council on Industrial and Applied Mathematics (ICIAM) Congress in Zurich, in July 2007. This award was introduced by the Association for Women in Mathematics (AWM) and European Women in Mathematics (EWM), a tribute to mathematician Olga Taussky Todd.

**Jozsef Solymosi** is the recipient of the 2008 André-Aisenstadt Prize, along with Jonathan Taylor (Université de Montréal). Concerning Dr. Solymosi's works, Each member of the selection committee was struck by the extraordinary efficiency and elegance of his results at the cutting edge of a new field, additive combinatorics (sometimes called arithmetic combinatorics). They appreciated the simplicity and deep insight in each of his works.

**Ronald van Luijk** is the winner of the 2007 G. de B. Robinson Award. The G. de B. Robinson Award was inaugurated to recognize the publication of excellent papers in the Canadian Journal of Mathematics and the Canadian Mathematical Bulletin and to encourage the submission of the highest quality papers to these journals. Dr. van Luijk received the award at the CMS's 2007 Winter Meeting in London, Ontario.

University of British Columbia's **Vinayak Vatsal** is the recipient of the CMS 2007 Coxeter-James Prize. The Coxeter-James Prize recognizes young mathematicians who have made outstanding contributions to mathematical research.

Dr. Vinayak Vatsal has made fundamental contributions to the Iwasawa Theory of elliptic curves, introducing profound techniques from ergodic theory into the subject and obtaining startling theorems on the non-vanishing of  $p$ -adic  $L$ -functions and  $\mu$ -invariants that had previously been unobtainable by more orthodox analytic methods. His 2002 Inventiones paper on the uniform distribution of Heegner points led to the complete solution of a fundamental conjecture of Mazur concerning such  $L$ -functions (now the Vatsal-Cornut theorem). In the words of his referees, these results have "transformed our understanding of the ranks of elliptic curves in towers of number fields.

Dr. Vatsal received the 2004 André-Aisenstadt Prize of the Centres de Recherches Mathématiques, the 2006 Ribenboim Prize of the Canadian Number Theory Association, and was an invited speaker at the 2006 International Congress of Mathematicians in Madrid.

Dr. Vatsal presented the 2007 Coxeter-James Prize Lecture at the CMS Winter Meeting hosted by the University of Western Ontario in December, 2007.

Two teams of UW undergraduates were declared Outstanding Winners in the 2007 Mathematical Contest in Modeling. Of the 949 international teams that participated in the MCM in 2007, 14 were named Outstanding Winners. U. Washington has had seven Outstanding Winners in the last six years.

**Zhen-Qing Chen** was inducted as a Fellow of the Institute of Mathematical Statistics (IMS) on July 30, 2007 at the IMS Annual Meeting in Salt Lake City, Utah.

Professor Chen received the award for research on Dirichlet form approach to Markov processes, reflected Brownian motion, stable processes, and for editorial services for the IMS-affiliated journals.

**Jim Morrow** has received the 2008 Deborah and Franklin Tepper Haimo Award for Distinguished College University Teaching of Mathematics from the Mathematical Association of America. The award is the most prestigious prize for higher-level math education in the United States.

Dr. Morrow is a former recipient of the PIMS Education Prize in 2005.

**Ginger Warfield** has been awarded the 2007 Louise Hay Award. This award, given by the Association for Women in Mathematics, recognizes outstanding achievement in mathematics education. The award was named for one of the association's founding members, a longtime math educator who was dedicated to students and had a lifelong commitment to nurturing the talent of young women and men in mathematics.



Recipients of the Leslie Fox Prize, Yoichiro Mori (UBC) (left) and Ioana Dumitriu (University of Washington)

**Ioana Dumitriu** (U. Washington) and **Yoichiro Mori** (UBC) are two of seven recipients of the Leslie Fox Prize in Numerical Analysis at Oxford University. Dr. Dumitriu received the award for her paper, "Toward accurate polynomial evaluation in rounded arithmetic," and Dr. Mori received the award for a paper he wrote on a convergence proof for the immersed boundary method.

# PIMS Conferences

## Statistical Distributions and Models: Assessment and Applications Simon Fraser University

April 19-20, 2007

by Charmaine Dean (SFU)

SFU's conference on *Statistical Distributions and Models: Assessment and Applications* on April 19-20 this year, held in conjunction with the PIMS 10<sup>th</sup> Anniversary lecture by David Brillinger, was dedicated to the areas of Model Assessment, Goodness-of-Fit and Directional Data, all areas of specialization of Professor Michael Stephens of Simon Fraser University whose 80<sup>th</sup> birthday was celebrated at the conference dinner. The opening talk, given by Professor David Brillinger of the University of California at Berkeley, was a PIMS Distinguished Lecture; it was titled "A unified approach to modelling trajectories" and described the use of stochastic gradient systems for modelling particles in motion. The ideas were applied to movements of animals such as elk, deer and seals and to the movement of ball in a soccer game.



David Brillinger  
(University of California,  
Berkeley)

Other speakers were: Louis-Paul Rivest, Université Laval, gave a talk entitled *A directional model for the determination of the anatomical axes of the ankle joint* in which data consisting of a sequence of rotation matrices were analyzed to determine the axes of rotation of the foot relative to the shank; Jerry Lawless, University of Waterloo, spoke on *Some Challenges in Assessing Goodness of Fit* and provided a thoughtful review of open problems in the area of goodness-of-fit; Richard Lockhart, Simon Fraser University, whose talk was simply titled *Michael and me* gave a largely historical talk touching on Michael Stephens' work and personal history; John Spinelli, B.C. Cancer Research Centre, reviewed work on problems of assessing models for discrete data such as commonly arise in large epidemiological studies in a talk titled *Goodness-of-fit for Discrete Data*; Federico O'Reilly, Universidad Nacional Autónoma de México, spoke on recent work on the development of exact tests of models by conditioning on sufficient statistics, and on methods to implement such tests, including Markov Chain Monte Carlo, in his talk: *Avoiding Asymptotics in Goodness-of-Fit*; John Petkau, University of British Columbia, discussed the measurement and evaluation of the progression of multiple



(l to r) Richard Lockhart, David Brillinger,  
Michael Stephens, CB Dean

sclerosis using the Extended Disability Status Scale in Stage III clinical trials and presented new methods to make more effective use of the longitudinal data arising in such trials. His talk was entitled *Evaluating Progression in Multiple Sclerosis Clinical Trials*; Ted Anderson, Stanford University, wrapped up the conference with a presentation on an important topic in Econometrics arising in simultaneous equation models: *Likelihood ratio tests in reduced rank regression and blocks of simultaneous equations*.

The conference was preceded by a meeting on the Thursday morning for new researchers in the Pacific Northwest, organized by Laura Cowen of the University of Victoria and Matias Salibian-Barrera from the University of British Columbia. Seventeen new researchers from the region attended. It featured a round table discussion devoted to providing guidance to the Statistical Society of Canada as to how that society might better serve new researchers, general discussion on the needs of new researchers in BC, on research and publishing, on teaching resources, on how to create professional links with industry, on the PIMS graduate student mathematics modeling camp and the industrial workshops, on ways to connect and be mentored by a senior researcher, and on ways new researchers in the region might more easily interact. Researchers found the meeting to be very useful, and the large turn-out reflected and emphasized the need for these sorts of networking activities.

The conference on Statistical Distributions and Models attracted 150 attendees including participants from universities and agencies throughout Canada, and from the USA, Spain and Mexico.

*Organizing Committee:* Richard Lockhart and Charmaine Dean



(l to r) Louis-Paul Rivest, Michael  
Stephens, Ted Anderson and Mrs.  
Anderson, Jerry Lawless, Richard  
Lockhart

# Workshop on Rock Mechanics and Logistics in Mining

## Santiago, Chile

February 26 to March 2, 2007

by Ivar Ekeland (UBC)

From February 26 to March 2, 2007, PIMS and the Centro de Modelamiento Matemático (CMM) organised a meeting on Rock Mechanics and Logistics in Mining in Santiago, Chile. CMM and PIMS invited MATHEON (Berlin), MASCOs (Melbourne) and MITACS to co-organize the workshop, and there were present with in important turnout. About 70 scientists were in attendance throughout the week, including eight representatives from PIMS, ranging from mathematics to mining engineering and economics. Highlights of the meeting included a speech by the Ministry of Industry, who stressed the importance of education and research in a resource-rich country like Chile, and a visit to El Teniente, the largest underground mine in the world.

El Teniente uses the technology of caving: instead of extracting the ore from the bottom up, like in open mining operations, or mining horizontally into the vein, in the traditional fashion, miners go under the vein and let in cave down in a controlled fashion, so that the work of detaching the blocks is done by gravity, and all the miners have to do is to carry them away. The ‘controlled’ part is of course the difficult one, and this is where rock mechanics come in: fracture enhancement and propagation, shape and growth of the cavern, all these lead to difficult problem in rock mechanics with inhomogeneous media. Designing the mine itself, with its network of caves, drawpoints and transportation galleries, is a logistics problem that is intimately connected with exploitation, and ultimately with the rock mechanics problem.

This is why PIMS and CMM are interested in mining: there is a beautiful interconnection between two widely different parts of mathematics, three-dimensional systems of partial differential equations on the one hand (also known as rock mechanics), and optimization on the other (also known as mathematical programming, or operations research). There is no more fertile ground for progress than cross-fertilization between two fields that at first seemed to far apart to communicate, and this is what has been happening in Chile. Engineers from CODELCO, the largest copper producer in the world, and HP-Billiton, also participated to share their experience and knowledge.

PIMS and CMM intend to continue and expand this mining initiative, within the framework of the international Collaborative Research Groups on Economics and Finance of Climate Risk and Natural Resources. The next step was the very successful summer school on “Energy Risk, Environmental Uncertainty and Public Decision Making” in Banff, which brought forward the economic and environmental aspects of the mining industry. Those who are interested and would like to contribute to this research program can contact the organizers of the CRG.

# Symposium on Kinetic Equations and Methods

## University of Victoria

April 27- 28, 2007

by Chris Bose (U.Victoria)

The Symposium on Kinetic Equations and Methods was originated as a ceremonial conference in celebration of the 10<sup>th</sup> founding anniversary of PIMS. It quickly became more: a first-class, state-of-the-art workshop including 10 speakers from the forefront of many research directions in kinetic theory and PDEs.

Senior administrators from U.Victoria (Martin Taylor, Vice-President Research Administration, and Tom Pedersen, Dean of Sciences) and Ivar Ekeland, Director of PIMS, were present to open the meeting on Friday morning. Their opening addresses were immediately followed by the lectures of José Carrillo and Fraydoun Rezakhanlou; as each lecturer had a full hour and the program was not overloaded, there was ample opportunity for questions and discussions, as well as further interaction after the lectures or during lunch breaks. The afternoon saw three more hour-long technical talks, and in the evening, all the speakers were invited to a dinner in the Wild Saffron restaurant.

Saturday saw a repetition of the lecture format, minus the formal introductions to open the meeting. The weather conspired beautifully, so a large group of lecturers and the audience joined an after-lunch



Participants in the Symposium on Kinetic Equations and Methods

walk to the nearby, sun-drenched Cadboro Bay Beach. The meeting closed at 5 p.m.

All participants agreed that this was a very rewarding scientific event. The approximately 30 participants were treated with a series of exceptional lectures, state-of-the-art methodologies, and a plethora of open questions from foundations to engineering applications.

# PIMS Mathematical Biology 2007 Summer Workshop

University of Alberta

May 1-5, 2007

by Drew Hanson (U.Alberta)

“It’s always hockey season in Edmonton”

Six words that have absolutely nothing to do with the Fifth Annual PIMS-MITACS Mathematical Biology Summer Workshop, only they’ll allow me, if you will, to make what I think is a fairly accurate analogy: the Math-Bio workshop was like playing in overtime. Maybe double-overtime. Maybe triple. More precisely, it was like winning in triple-overtime. It was May 1<sup>st</sup>, and the twenty-one of us had all just finished our school year. After three or four weeks of complete focus on exams, and eight months of overusing our brains, we were done. Some in our group had in fact just graduated, a feat surely worthy of a week or two of nothing but rest. We had listened and learned and studied for exactly as long as we had planned, and our budgeted energy for such tasks had been exhausted. However, because we were interested, because we were curious, and, well, because everyone else seemed to be doing it, we all gave it two more weeks. At times during the ten days of classes, it seemed to each of us like we had reached and passed our capacity to take in any more information. At times it seemed like we were getting nowhere, like our projects would contain nothing more than a title page and a list of “future areas of study.” By May 11th, however, we had produced ten fascinating projects covering some very interesting questions, and using some very interesting methods, in the field of Mathematical Biology. Thanks to four instructors, a guest lecturer, about 15 volunteers, and an inexplicable motivation to learn even more, we all accomplished something very rewarding, and went home feeling very fulfilled.

In the first week of the workshop, we had two lectures covering one topic each day, split in the middle by a lunch break and a seminar in Maple. The lectures focused on teaching the thought process behind forming a model. Examples of existing models were given using both discrete-time and ordinary differential equations, as well as stochastic and non-stochastic approaches. For each, we were taught about studying the stability of the model at different fixed-points, as well as the concept of  $R_0$ , the basic reproduction number, which refers to how many diseased individuals a single diseased individual can create. Finally, we had one lecture about parameter estimation. By the end of this first week, we were all quite worn-out, but not one of us realized how much we had learned.

The second week was all about applying everything we’d been taught. Almost by brute force,

the professors were able to put us into project groups that gave us an opportunity to work both on a project we were interested in, and with a peer from a different field of study. We discussed that as valuable a project as any would have been to write a computer program to form the groups for workshops to come, but justifying that as a project in Mathematical Biology was questionable, so we moved on as planned. A study of hoof and mouth disease using an SIR model was used to determine the level of vaccination required to eliminate the disease from a cow population. The possibility of using bioengineered macrophages to kill tumour cells, and similarly, the possibility of introducing transgenic malaria-resistant mosquitoes into a diseased population were both looked at. Two separate groups used disease-dynamics to study the spread of rumours, with some conflicting conclusions. As well, the effect of genetically superior, farmed salmon escaping into the wild was modeled, with some very interesting initial results. Control of the heart via electrical impulses was the subject of a model using nonlinear dynamical systems, and one group used a gravity model, which uses the principle that bigger objects are more attractive, to model the dispersal of a beetle population over a number of years. Lastly, a group looked into some of the patterned mathematical elements of what occurs during sleep.

The summer workshop was an incredible experience. We met people from similar and different fields of study, from all over Canada, from Germany, Costa Rica, and Portugal. We worked with some world-renowned professors in the field of Mathematical Biology, and were helped by some graduate students who are sure to be the same in a short time. And, challenged by the constraints of time and experience, each group produced very exciting and relevant results using our newfound skills. I have no doubt the every participant in the workshop left with new friends, new interests, and a vastly increased understanding of this branch of mathematical sciences.



Participants in the PIMS Mathematical Biology 2007 Summer Workshop

# First North American Regional TIES Meeting University of Washington

June 19-21, 2007  
by Peter Guttorp (U.Washington)

The first North American regional meeting of TIES took place in Seattle June 19-21, 2007. Ninety registered participants enjoyed a variety of talks, many geared towards the conference theme of “Climate change and its environmental effects: monitoring, measuring, and predicting.”

The conference started in the afternoon of the first day with a session on inference for mechanistic models, with Tilmann Gneiting, Mark Berliner and Derek Bingham as speakers. The first keynote address followed, with Paul Switzer presenting “Regional time trends in climate model simulations.” The opening mixer, with five poster presentations, concluded the first day’s activities.

The second day saw two invited sessions on Monitoring the environment and biota on landscape to continental scales. The speakers were Jay Breidt, Jason Legg, Gretchen Moisen, Don Stevens and Mevin Hooten. An invited session on Paleoclimatic temperature reconstruction had talks by Edward Cook, David Schneider and Bo Li. Elizabeth Shamseldin, Georg Lindgren and Slava Kharin were invited speakers in a session on Assessing trends in extreme climate events. In parallel, a special session by conference sponsor National Oceanic and Atmospheric Administration included Bill Peterson, Peter Lawson, Kerym Aydin and Lisa Crozier. The conference dinner took place in one of the dorms, but the catered food was several levels above regular dorm food!

The final day had a morning invited session on Agroclimate risk assessment with Nathaniel Newlands, Jim Ramsay and Nhu Le. After the coffee break two parallel invited sessions took place: one on The role of statistics in public policy with Paul McElhany, Tanja Srebotnjak and Marianne Turley, and the other on Measuring biodiversity and species interaction, having speakers Andy Royle and Emily Silverman.

The second keynote address was by TIES president David Brillinger, and was entitled “Probabilistic risk modeling at the wildland-urban interface: the 2003 Cedar Fire.” The final invited session was about Forests, fires and stochastic modeling. Speakers were Mike Flannigan, Haiganoush Preisler and Steve Taylor.

Throughout the program there were contributed sessions on Inference for mechanistic and stochastic models; Spatial methods; Methods in ecology; Forest fires, remote sensing and stochastic models; and Climate.

The program committee consisted of Peter Guttorp, University of Washington (chair), Ashley Steel, NOAA Fisheries Seattle, Emily Silverman, USFWS Maryland, Joel Reynolds, USFWS Alaska, Eliane Rodrigues, UNAM, Mexico City and Jim Zidek, University of British Columbia Vancouver BC. The local organizing committee was Paul Sampson, University of Washington, together with Guttorp and Steel.

The conference had financial support from the US National Science Foundation, the Pacific Institute of Mathematical Sciences, and the US National Oceanic and Atmospheric Administration. In particular this enabled us to fund the travel for many of the 26 students at the meeting.

We are hopeful that there will be another North American Regional Meeting, perhaps in 2009. Don Stevens of Oregon State ([stevens@science.oregonstate.edu](mailto:stevens@science.oregonstate.edu)) is looking for volunteers to help organize it.

# Intuitive Geometry Workshop and Intuitive Geometry Day University of Calgary

Aug. 31 - Sept. 3, 2007  
by Ferenc Fodor (University of Szeged, Hungary)

This two-day workshop was organized to provide a much desired opportunity to share research findings in the interconnected fields that are represented in Intuitive Geometry. The term Intuitive Geometry was coined by László Fejes Tóth to denote those geometric disciplines in which the unifying theme is that their problems themselves can be explained fairly easily, even to an advanced high school student, however, the solution of these problems require difficult and very deep methods of modern mathematics. This Workshop is also part of a series of Intuitive Geometry conferences the first of which was organized in 1975 in Tihany, Hungary, and the last one was in 2000 in Balatonföldvár, Hungary. This workshop was the sixth such meeting. The Intuitive Geometry Workshop was immediately followed by the Intuitive Geometry Day in Calgary held at the Department of Mathematics and Statistics of the University of Calgary. The Intuitive Geometry Day was a direct continuation of the Intuitive Geometry BIRS workshop. Its main purpose was to provide an extension to the BIRS event and thus make attendance of the workshop more desirable to colleagues from overseas. In this regard, the event was a great success, out of the 30 participants 11 were from outside North America. The 30 participants of the meetings gave 24 high quality research talks on their recent results of which 16 were 30-minute and 8 were 20-minute presentations. Subject of talks covered the broad areas of general convexity, iterative geometric processes, the theory of packing and covering both in Euclidean and hyperbolic spaces, polytopal approximation of convex bodies, Minkowski geometry, combinatorial geometry, the theory of geometric transversals, extremal problems for convex sets, and abstract and convex polytopes.

The workshop was a resounding success, it brought together researchers from many different fields of Geometry, and among them, three advanced graduate students and several postdoctoral fellows. New collaborations among participants are already noticeable, especially among the graduate students and postdocs.

Results presented at the Workshop and the Intuitive Geometry Day in Calgary will be published in a special Intuitive Geometry volume of the journal *Periodica Mathematica Hungarica*. In summary, the future directions for research in Intuitive Geometry are plentiful and the area is very much alive being a central part of modern geometric research.

The Intuitive Geometry Day in Calgary was generously supported by the Pacific Institute for the Mathematical Sciences, the Faculty of Science, and the Department of Mathematics and Statistics of the University of Calgary.

*Organizing committee:* T. Bisztriczky (University of Calgary, Canada), G. Fejes Tóth (Alfréd Rényi Institute of Mathematics, Hungary), F. Fodor (University of Szeged, Hungary), W. Kuperberg (Auburn University, U.S.A.)



# Tropical Multiscale Convective Systems: Theory, Modeling, and Observations Summer School and Workshop

University of Victoria

July 30 to August 3, 2007

Climate modelers, observationalists and theorists from across North America gathered with graduate students and post-doctoral fellows at the University of Victoria on July 30 to August 3, 2007, to discuss developments in tropical multiscale convective systems research. The summer school was an initiative of the PIMS Collaborative Research Group on Mathematical Problems in Climate Modeling.

With an increasing global focus on climate change and the science underlying climate processes, the CRG on Mathematical Problems in Climate Modeling is a multidisciplinary collaboration between mathematicians and earth and ocean scientists to examine outstanding problems in climate modeling and numerical weather prediction, with particular emphasis on multiscale processes in the tropics. The CRG aims to bridge the gap between idealized models and the general circulation models currently used by government forecasters.

The conference at UVic focused on tropical multiscale convective systems. Historically, the field of tropical meteorology was advanced by the discovery by Madden and Julian in 1971 that the dominant component of intraseasonal variability in the tropics is a 40- to 50-day oscillation (now known as the Madden-Julian oscillation (MJO)). In 1988, Nakazawa suggested that the tropical intraseasonal oscillations are actually space/time envelopes of organized clusters and super-clusters of convective clouds occurring on shorter scales.

While the MJO envelope propagates eastward, the embedded clusters and super-clusters move at faster speeds in both east and west directions. Studies identified these clusters and superclusters as the moist equivalents of the linear shallow water equatorially trapped waves, while the MJO has no linear equivalent. Organized convection and convectively coupled waves in the tropics have a significant impact on midlatitude weather and climate through atmospheric and oceanic teleconnection patterns. While a broad range of mechanisms has been proposed to explain the MJO, it is poorly represented in contemporary general circulation models (GCMs).

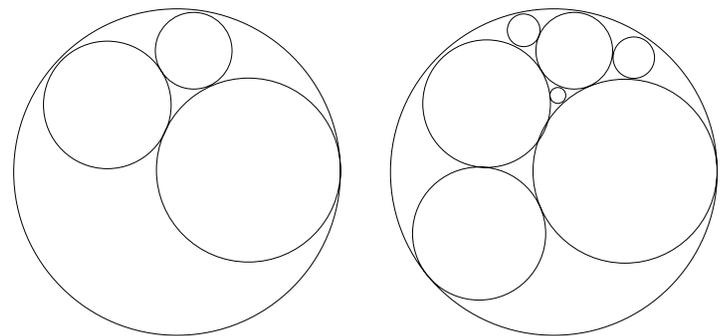
The three-day summer school, followed by a two-day workshop, focused on these tropical multiscale convective systems. In recognition of the complex and multiscale nature of the issue, lecture topics covered the spectrum from cloud microphysics through convective organization to global-scale dynamics.

## Speakers

**Philip Austin** (UBC) presented an overview of dynamical and thermodynamic aspects of atmospheric convection in the tropics.

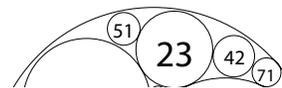
**Wojtek Grabowski** (National Center for Atmospheric Research) spoke on cloud-resolving modeling (CRM), as many research groups around the world are working on global cloud resolving and superparametrization models as alternatives to represent organized convective features.

**Boualem Khouider** (UVic) discussed waves and instabilities in idealized model convective parameterizations, with the aim of assessing some different parameterization closures currently in use.



Generation 1

Generation 2



Participants in the Tropical Multiscale Convective Systems summer school and workshop

**George Kiladis** (National Oceanic & Atmospheric Administration) spoke on observations of convectively coupled waves starting from the power spectra of Wheeler and Kiladis, using satellite data to illustrate organized convective clusters and super-clusters, including the MJO.

**Norm McFarlane** (CCCma) lectured on the complex subject of convective parameterizations covering both mass flux and adjustment (Betts-Miller) based schemes.

**Andrew Majda** (Courant Institute, NYU) discussed the theory of equatorially trapped waves, beginning with an emphasis on the importance and complexity of tropical meteorology.

**Mitchell Moncrieff** (NCAR) discussed organized mesoscale convection, making the analogy between organized convective systems and coherent structures in a turbulent flow (both involving upscale transport).

**Cecile Penland** (NOAA) discussed “The Multiple Scales of El Niño,” starting with an overview of El Niño-Southern Oscillation (ENSO) phenomenology and developing a theory of ENSO prediction based on the theory of linear inverse modeling.

**John Scinocca** (Canadian Centre for Climate Modelling and Analysis) presented an introduction to GCMs.

**Knut von Salzen** (CCCma) focused on the subject of cloud microphysics, emphasizing the role of aerosols for the formation of cloud droplets, presenting observations to illustrate effects of aerosol/cloud interactions on climate, including interactions of microphysical and cloud dynamical processes, which remain in general poorly understood.

The two-day workshop featured 20 presentations of new research by meeting participants. Discussions covered observational and theoretical features of convection, processes involving the extratropics, and wave and instability dynamics.

Lecture materials can be downloaded from

<http://pims.math.ca/science/2007/07sstmcs/>

# International Graduate Training Centre in Mathematical Biology - First Graduate Research Summit

## University of British Columbia

September 28-30, 2007

by Hannah McKenzie (U.Alberta)

The First Graduate Research Summit, hosted by the **International Graduate Training Centre in Mathematical Biology (IGTC)**, took place September 28-30, 2007, at the University of British Columbia. The summit was attended by more than 50 graduate students, postdocs and researchers from the U.Alberta, UBC, U.Victoria, U.Utah and Simon Fraser University. Daniel Coombs (UBC) opened the summit on Friday night, speaking about coupling within-cell, within-host, and within-population dynamics in infectious disease models to further understand levels of competition and selection. Following the first plenary talk, participants mingled at Thea's Penthouse in the Graduate Student Centre, to enjoy delicious food.

On Saturday morning, Mark Lewis (U.Alberta) gave the second plenary talk about the spatial dynamics of emerging wildlife diseases, particularly West Nile virus and sea lice (a parasite of salmon). The following sessions on Saturday were filled with short talks addressing diverse and interesting topics in mathematical and statistical biology, three of which are highlighted below.

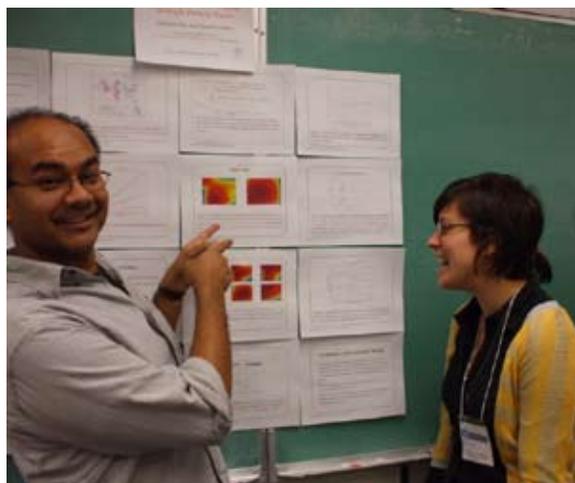
The first session focused on applications to cellular biology. Jun Allard (UBC) presented a model of the actin-like MreB helix in prokaryotes. MreB is involved in cell growth and is thought to govern protein trafficking within the cell. Allard's model predicted a relationship between pitch of the helix, thickness of the helical cables, and total abundance of MreB in the cell. This relationship provides a way of deducing properties of the helix that are experimentally difficult to measure.

The focus of the second session was mathematical ecology. Raluca Eftimie (U.Alberta) spoke about the important role of different animal communication mechanisms for the formation of complex spatial group patterns. Numerical analysis of her hyperbolic model for animal group formation and movement revealed a wide range of spatial and spatio-temporal patterns. Some of these are classical patterns such as traveling waves

or stationary pulses, while others, such as breathers and zig-zag pulses are completely new.

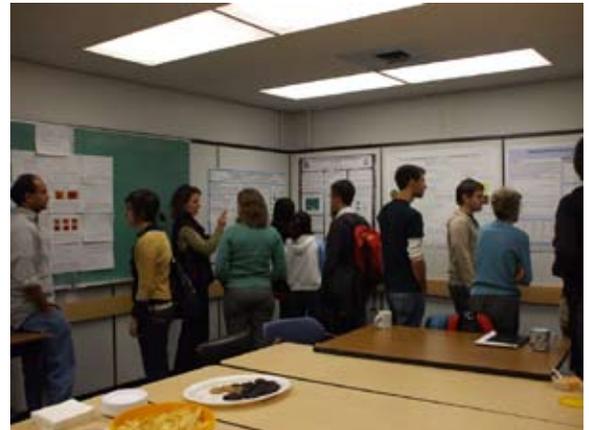
In her talk in the final session, Linghong Lu (U.Victoria) focused on Glass networks, which are used to model gene networks. Networks of interacting genes have complicated interactions in which the promotion or repression of one gene's expression may depend on the activity states of several other genes. It is useful in systems biology to understand the possible types of dynamical behaviour of certain classes of gene networks. Linghong described some such structural principles that she derived, allowing periodic orbits and certain types of complex dynamical behaviour. An exciting day of science was completed with an evening poster session.

Sunday morning found summit partici-



Dodo Das ( UBC) and Erin Prosk at the poster session

pants joining in a professional development session led by Jack Tuszyński (Cross Cancer Institute, U.Alberta) and Pauline van den Driessche (U.Victoria). Jack and Pauline discussed their academic career path with students and postdocs, and entertained many questions ranging from the characteristics of a successful graduate student to how to attain a tenure-track position. In the afternoon, some participants braved the rain for a hike along the beach, while others explored downtown Vancouver. As the weekend drew to a close, students, postdocs, and researchers headed back to their home universities with new ideas, contacts, and a sense of excitement about mathematical biology.



Participants at the poster session



The participants of the research summit

# New Collaborative Research Groups 2008-2011

A PIMS Collaborative Research Groups consists of researchers with common research interests and with a common desire to collaboratively develop some aspects of their research programs.

PIMS CRGs develop permanent research and training networks, establish lasting links between geographically separate groups of researchers, provide sustained mentorship for postdoctoral fellows and students over two or three years, and enhance the international profile of Canadian researchers.

CRGs create critical mass that substantially enhances training programs at all levels. The pooling of PIMS support with other sources and the joint planning of resource allocation allows the CRGs to support a large number of PDFs and graduate students and will create new research opportunities for these young scientists, including exchanges, joint supervision, and summer schools.

## CRG in Partial Differential Equations

Partial Differential Equations is a large subject with a history that dates back to Newton and Leibniz. They form the basis for many mathematical models in the sciences and in economics, yielding such famous equations as the Euler and Navier Stokes equations and Schrödinger's and Einstein's equations. Moreover, the subject is far more than just a mathematical tool to address physical and economic phenomena: PDEs have guided and created the fields of nonlinear functional analysis, harmonic analysis, optimization and the modern calculus of variations, and have had a major impact on the field of geometry. The latter was recently highlighted by the role of Ricci flow in the eventual proof of the celebrated Poincaré Conjecture.

This diversity is reflected in the research of many faculty members across the PIMS universities and this Collaborative Research Group will provide the means for bringing together faculty and postdoctoral fellows from different PIMS universities; and sponsoring graduate and post doctoral courses and minicourses which will provide access to a wide audience at PIMS universities, as currently there is not the critical mass to offer such course at the majority of the PIMS universities. The CRG will focus on the key areas of:

- geometry and analysis of dispersive equations;
- regularity for solutions of certain fundamental equations (Navier-Stokes, harmonic measures, the infinity Laplacian);
- the role of hyperbolic problems in traffic flow, kinetic theory and the material sciences;
- modern approaches to asymptotic analysis in the calculus of variations and applications;

- general variational principles;
- universal inequalities in relevant function spaces; and
- the role of game theory and stochastic methods in certain elliptic and parabolic equations.

The period of concentration will be inaugurated in the winter of 2008 with a joint PIMS-CNRS minisymposium to celebrate the new partnership between the two institutions. The PDE theme is fitting for this event in view of the presence of a substantial number of PDEs among the CNRS researchers at the PIMS universities. Two Pacific North West conferences will occur every year; at U.Victoria (Winter 2008), UBC (Fall 2008), U.Washington (Winter 2009), and SFU (Fall 2009).

As a flagship event that will hopefully cement our CRG and that will make its role visible and lasting on the international level, we propose a Thematic Program in PDE for July and August of 2009. The program will consist of six week-long workshops with focuses ranging from well-established and fundamental problems (eg. regularity for Navier Stokes) to fields which are only emerging today (eg. connections between stochastic games and elliptic and parabolic PDE). All these workshops will bring in both short and long term speakers/visitors who are at the forefront of their fields. We expect this thematic summer to have a central role in attracting young researchers (from graduate students to pdfs) to the PIMS universities.

*CRG Leaders: Rustom Choksi (SFU), Reinhard Illner (UViC), Nassif Ghoussoub (UBC)*

## CRG in Bayesian Modeling and Computation for Networks

This Collaborative Research Groups focuses on Bayesian methods for network analysis, paying special attention to model design and computational issues of learning and inference. Bayesian inference is an approach to statistics in which all forms of uncertainty are expressed in terms of probability. Non-Bayesian approaches to inference have dominated statistical theory and practice for most of the past century, but the last two decades have seen a reemergence of Bayesian statistical inference. This is mainly due to the dramatic increase in computer power and the availability of new computational tools, including variational techniques, Markov chain Monte Carlo (MCMC) and sequential Monte Carlo (SMC). Bayesian modeling has become common practice as it provides a powerful method for coping with very complex stochastic domains, including networks. Networks are widely used to represent data on relations between interacting actors or nodes. Among many things, they can be used to describe social networks, genetic regulatory networks, computer networks, and sensor networks. In these settings, traditional independence assumptions are blatantly inappropriate; the structure of relationships between the data must be taken into account. As a result, there has been increasing research developing techniques for incorporating network structures into machine learning and statistics. This collaborative research group will bring together researchers working on Bayesian modeling for networks from different communities, thereby fostering collaborations and intellectual exchange. Our hope is that this will result in novel modeling approaches, diverse applications, and new research directions. In particular we will focus on three main problems: social networks, regulatory networks and sensor networks. Even though the three problems share many similar features, both in terms of modeling and computation, they are usually treated separately.

*CRG Leaders: Raphael Gottardo (UBC), Paul Gustafson (UBC), Lurdes Inoue (UW), Adrian Raftery (UW), Tim Swartz (SFU)*

# Pacific Institute for the Mathematical Sciences Upcoming Activities



## **Disease Dynamics 2008**

University of British Columbia, April 3–5, 2008

## **ABC Algebra Workshop**

Simon Fraser University, April 12–13, 2008

## **Regulators and Heights in Algebraic Geometry**

University of Alberta, April 12–16, 2008

## **National Institute in Complex Data Structures Workshop**

University of British Columbia, April 24–25, 2008

## **Waves in Atmosphere and Ocean Workshop**

Simon Fraser University, April 25–26, 2008

## **2008 University of Alberta Summer School on Mathematical Modeling of Infectious Diseases**

University of Alberta, May 1–11, 2008

## **Canadian Young Researchers Conference in Mathematics and Statistics 2008**

University of Alberta, May 2–4, 2008

## **Sixth Annual PIMS Mathematical Biology Summer Workshop: Mathematics of Biological Systems**

University of Alberta, May 6–16, 2008

## **Number Theory Day at the University of Lethbridge**

University of Lethbridge, May 7, 2008

## **IGTC Graduate Summer School in Mathematical Biology**

University of British Columbia, May 11 – June 11, 2008

## **Mathematical Interests of Peter Borwein**

Simon Fraser University, May 12–16, 2008

## **Eighth Algorithmic Number Theory Symposium ANTS-VIII**

Banff Centre, May 17–22, 2008

## **Lie Theory and Geometry: the Mathematical Legacy of Bertram Kostant**

University of British Columbia, May 19–24, 2008

## **Western Canada Linear Algebra Meeting (WCLAM) 2008**

University of Regina, May 30–31, 2008

## **11<sup>th</sup> PIMS Graduate Industrial Mathematics Modelling Camp**

University of Regina, June 9–13, 2008

## **2008 PIMS Summer School in Probability**

University of British Columbia, June 11 – July 8, 2008

## **12<sup>th</sup> PIMS Industrial Problem Solving Workshop**

University of Regina, June 16–20, 2008

## **Workshop on Variational Methods and Nash-Moser**

University of British Columbia, June 16–22, 2008

## **Math Finance Summer School**

University of British Columbia, June 30 – July 11, 2008

## **Algebraic Aspects of Association Schemes and Scheme Rings**

University of Regina, July 8–11, 2008

## **15<sup>th</sup> Canadian Undergraduate Mathematics Conference (CUMC 2008)**

University of Toronto, July 9–12, 2008

## **Summer School in Stochastic and Probabilistic Methods For Atmosphere, Ocean, and Climate Dynamics**

University of Victoria, July 14–18, 2008

## **Workshop on Transport, Optimization, Equilibrium in Economics**

University of British Columbia, July 14–20, 2008

## **Stochastic Parameterisations in Atmosphere and Ocean Models Workshop**

University of Victoria, July 21–25, 2008

## **2008 Society of Mathematical Biology Annual Meeting**

University of Toronto, July 30 – August 2, 2008

## **Summer School on Particles, Fields and Strings**

University of British Columbia, August 4–15, 2008

## **Northwest Dynamics Symposium III**

University of Victoria, August 5–9, 2008

## **Mathematical Graphics and Visualization Workshop**

Simon Fraser University, August 7–15, 2008

## **Similarity: Generalizations, Applications and Open Problems**

University of British Columbia, August 10–15, 2008

## **The Second International Conference on Information Theoretic Security (ICITS 2008)**

University of Calgary, August 10–13, 2008

## **International Conference on Quantum Communication, Measurement, and Computing 2008**

University of Calgary, August 19–24, 2008

## **Third Canadian Summer School on Communications and Information Theory**

Banff Park Lodge, August 21–24, 2008

## **Geometric Analysis: Present and Future**

Harvard University, August 27 – September 1, 2008

## **Oceanic Gravity Waves**

University of Washington, Fall, 2008

## **International Conference on Infinite Dimensional Dynamical Systems**

York University, September 24–28, 2008

## **2008 Fall Western Section Meeting of the American Mathematical Society**

University of British Columbia, October 4–5, 2008

The 2008 Einstein Lecture to be given by Freeman Dyson (Institute for Advanced Study in Princeton) on October 4, 2008 at the University of British Columbia, Vancouver. The title of his lecture will be *Birds and Frogs*.

## **WIN – Women in Numbers**

Banff Centre, November 2–7, 2008



# 9<sup>th</sup> Annual PIMS Elementary Grades Math Contest (ELMACON)

University of British Columbia

May 5, 2007

by Melania Alvarez; photographs by Meng-Chieh Wu

The 9<sup>th</sup> Annual PIMS Elementary Grades Math Contest (ELMACON), held on May 5, 2007, was very successful. A Math Mania Event was held at the same time, with parents and siblings participating as well as the contestants. ELMACON is open to B.C. Lower Mainland students in Grades 5 to 7. It gives them a chance to experience mathematics as an exciting sport.

ELMACON consists of three rounds starting with the written component, the Sprint and Target rounds. The top 10 students in each grade go on to the Countdown round where contestants 'duel' against each other. It starts with the 9<sup>th</sup> and 10<sup>th</sup> ranking contestants, and the winner of that contest goes on to 'duel' the 8<sup>th</sup> place holder. So the contestant who is ranked 10<sup>th</sup> after the first two rounds has the potential of winning the contest by beating the nine contestants ahead of him or her. The dueling consists of answering math questions against the clock and sounding a buzzer.

A total of 364 students participated in the competition: 141 in Grade 5, 123 in Grade 6, and 100 in Grade 7.



The winners of 2007 ELMACON

PIMS would like to extend a huge thank you to Ilan Keshet, Joshua Keshet, Andrew Adler, Cary Chien, Klaus Hoeschmann, Natasa Sirotic, Ilija Katic, Jordan Forseth, Olivia Mak, and Robyn Massel, for preparing the contests' questions, overseeing the marking of the tests and proctoring the contest. We want to recognize Marlowe, Mar Ness and Yvonne Diamond, the staff of the Mathematics Department, and Ken Leung, Chee Chow, Breeonne Baxter and Danny Fan, the PIMS and BIRS staff, for their great support. We want to thank Meng-Chieh Wu for taking such great pictures of both events.



The Parity Cups game at Math Mania



Participants busy during the Sprint and Target rounds



A probability game at Math Mania



A Math Mania participant plays "Twenty Four"



Math Mania participants play the Sorting Game

## PIMS 2007 Education Prize Awarded to Sharon Friesen

Sharon Friesen, Associate Professor at U.Calgary, was awarded the PIMS 2007 Education Prize in June, 2007. Dr. Friesen is the first professional educator to whom the prize has been awarded.

The PIMS Education Prize recognizes a member of the PIMS Community who has made a significant contribution to education in the mathematical sciences, enhanced public awareness and appreciation of mathematics, and fostered communication among organizations involved in math training.

Dr. Friesen presented a talk on *Improving Teaching and Learning of Mathematics: Galileo Educational Network*. The Galileo Educational Network in collaboration with a number of mathematicians, has been working to provide K-12 teachers with professional learning opportunities related to improving mathematics teaching and learning. Members of the Galileo Educational Network provide teachers with professional support through: professional learning sessions, Math Fairs, Lesson Study and classroom-based context-specific planning sessions with individual teachers.

Dr. Friesen is co-founder and president of the Galileo Educational Network. She consults on a wide range of teaching and learning topics related to curriculum reform and school improvement.



Sharon Friesen (U.Calgary)

## FAME 2007 a Great Success

by Wendy Swonnell (Lambrick Park Secondary School, Victoria)

FAME (Forever Annual Mathematics Exhibition) is a mathematics fair held every year in the Greater Victoria School District. It is open to any student from Kindergarten to grade 12. FAME 2007 was held at S.J. Willis Auditorium on Tuesday, April 17, from 12:00 p.m. until 6:00 p.m. This was the 10<sup>th</sup> year for FAME. There were 12 senior entries, 24 middle school entries, and 36 elementary entries, and a total of 144 students competed. Eleven schools were involved and there were 14 Distinction awards (scores 90%+) presented.

The winning schools (top three scores per school) were Campus View at the elementary level, Arbutus at the middle school level and St. Margaret's at the secondary level. The trophies presented to each winning school were paid for by PIMS and PIMS also pays for the medals, rosettes and ribbons presented to the top entries. The students have to present their projects to a pair of judges, and it is always a delight to see and hear the young students present explanations of math ideas in their own way.

In 2008, FAME will be held on April 15. We are expecting a record number of entries and the quality continues to improve each year.

## Alberta Colleges Conference University of Alberta

May 4, 2007

by Jack Macki (U.Alberta)

The Alberta Colleges' Conference was held May 4, 2007 at the University of Alberta, in conjunction with the North-South Universities' Conference on May 5.

Peter Zizler (Mount Royal College) described a research project on spatial crime risk assessment based on an analysis of previous crime locations. Manny Estabrooks (Red Deer College) gave a presentation on innovative activities at the college aimed at helping students survive calculus. They include Doug Girvan's Calculus Prep, offered in August to incoming students, and Conrad Ferris' Completion Seminar, which allows students who fail a calculus course to take a special additional seminar/course, with web-based assignments and exams using eGrade, to raise their mark to a pass. Much of the material in these courses could be shared province-wide. Results from these efforts have been encouraging.

Prof. Estabrooks went on to discuss eLearning in general. There is a problem with the cost of private systems, and often they are tied to a particular text. His experience indicates that eLearning only works well for some students, while classroom learning works best for others. He is developing a proposal for PIMS funding of an initiative to evaluate several eLearning systems - among them commercial systems, Maple, and WebWork.

Cristina Popescu Anton presented her observations on evaluation in an introductory statistics course. She identified some of the major difficulties faced by the students: lack of ability with basic common sense comparison of decimals (1.4 is less than 1.22 because 4 is less than 22), fractions and proportions, solving equations or small systems, associating symbols with unknowns in word problems, counting all outcomes in probability problems.



She very carefully analyzed the reasons students missed the correct analysis of a deer range distribution problem, and then changed her approach based on how she analyzed the students' thinking processes.

Kathy McCabe, Stella Shrum and Merla Boland presented upcoming changes in the Alberta High Schools Mathematics Curriculum. One major driving force behind these changes is the proposed Western and Northern Canadian Protocol, involving the education ministers from Manitoba, Saskatchewan, Alberta and British Columbia and the three Territories. Implementation is expected over the period 2010-2012. High school mathematics will be three sequences: Math for the Sciences, preparation for university calculus; Fundamentals of Math, leading to non-calculus-based programs at the post-secondary level; and Trades and Workplace Math. Grade 10 math for the first two sequences is common. Transfer between these two should also be possible in Grades 11 and 12. The Fundamentals Course contains inductive and deductive reasoning and some logic and set theory, while the Math for Sciences does not. This is a matter for concern, and the lack of reasoning and proof in the pre-calculus sequence was commented on.

Jack Macki (U.Alberta) described the approval by the PIMS Board of PIMS Education Associates, aimed at the colleges. Education affiliates would work with PIMS on education and outreach projects. The program will be launched in the upcoming year.

Pacific Institute for the Mathematical Sciences

# Industrial Short Course

## Monte Carlo Methods for Financial Modelling

Monte Carlo methods involve the use of randomly-generated scenarios to compute results. They are a simple yet powerful tool that can provide solutions to large-scale problems in a fraction of the time of alternative approaches. The term 'Monte Carlo methods' was coined by physicists working on the Manhattan Project in the 1940s, but the methods have truly come into their own over ensuing decades with the development of computers, and are used now in diverse fields such as computer graphics, reliability engineering, environmental modelling, nuclear physics, molecular chemistry and operations research.

*Intended Participants:*

This course has been designed to provide an introduction to the basics of Monte Carlo methods in financial modelling, and will cover some recent developments. It is aimed at those seeking to use Monte Carlo techniques to obtain fast and accurate option prices and greeks, or to assess the risks of complex portfolios, or simply to gain a better understanding of what goes on 'under the hood' of commercial risk-management packages. While a financial context will be explored using Monte Carlo methods, any individual with an interest in using this modeling approach will find this course practical. A basic familiarity with option pricing and risk management will be helpful, no computer programming experience will be assumed.

Instructor:  
Tony Ware (U.Calgary)

Dates:  
June 24–25, 2008

